THE

ITHMETICAL VADE MECUM;

OR,

POCKET REFERENCE.

DESIGNED AS A WORK OF PRACTICAL UTILITY FOR THE TEACHER, THE MAN OF BUSINESS, AND THE MATHEMATICAL STUDENT.

BY I. N. WILCOXSON;
THREE SPRINGS, HART COUNTY, KENTUCKY.

LOUISVILLE, KY:
MORTON & GRISWOLD, PRINTERS.
1858.

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TO

REV. J. P. MURREL,

PRINCIPAL OF CAMDEN SEMINARY,

IN TOKEN

OF OUR HIGH APPRECIATION OF HIS LIFE AND CHARACTER

AS A

TEACHER OF YOUTH;

AND TO

ED. PORTER THOMPSON,

MY YOUNG FRIEND AND FELLOW-STUDENT,

THIS LITTLE WORK IS

RESPECTFULLY AND

FRATERNALLY

INSCRIBED,

BY THE

AUTHOR.

PREFACE.

It may be thought that the great number of Arithmetical works, now before the public, renders it unnecessary to multiply them; but in adding another to the list, we deem it sufficient apology to remark, that the importance of mathematical science renders it justifiable in any one to present to the public every improvement that will enable calculators to arrive, with greater facility, from premises to conclusions. The increasing interest manifested in the cause of Education, and the labor of Rev. J. P. Murrel, Principal of Camden Seminary, together with that of the author of these pages, for many years, in abridging and simplifying the common methods of calculation, are sufficient inducements for the publication of the result of those efforts. The method of statement given, is better adapted to all questions that may arise, in business transactions, than any heretofore offered.

We have arranged the common rules of Practical Arithmetic — Simple and Compound Proportion, Practice, Interest, Discount, Barter, Percentage, and those for all kinds of Mensuration — under the general head of *Cause ana Effect;* and it will be perceived that the method

of reasoning employed, adapts itself to the most common capacity, and is applicable to the solution of all questions that may arise under any of those individual heads. As the work is designed for the teacher, the business man, and the advanced mathematical student, and as our object has been to illustrate the nature and principles of calculation, we have given only a limited number of problems; deeming it unnecessary to give more than a sufficiency to illustrate the rules, as competent teachers can readily give examples under every head, and to the business man questions will naturally arise.

It will be observed that we have mostly used abstract numbers; but as it is presumed that those who use the work will have previously become acquainted with both the Simple and Compound rules of Addition, Subtraction, Multiplication, and Division, they can adapt the rules here given to the solution of all questions involving different denominations.

Indulging the fond hope that this little work may prove itself worthy the consideration of those for whom it is designed, we now submit it to a generous public.

I. N. WILCOXSON.

Hart County, Kentucky, August, 1855.

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EXPLANATORY ARTICLES.

ARTICLE 1. In placing numbers on the line, place the individual numbers directly under each other. If fractions, place the numerator where you would place it were it a whole number, with its denominator on the opposite side. The solution of fractional sums in Multiplication and Division will then be nothing more than those in whole numbers.

ART. 2. Observe that we cannot add, subtract, multiply, or divide numbers of different denominations, without first reducing them to the same denomination ; also, that no ratio can exist between different denominations, but the abstract numbers representing them may have a ratio.

ART. 3. To render verification easy, observe that subtraction is the *converse* of addition, and division of multiplication.

ART. 4. A *unit* is the basis of *all* work ; the point from whence the analytical student reckons with facility ; for the value of a fraction is decreased when multiplied by itself, while any number greater than one is increased. Unity is the result of any fraction divided by itself ; and, $1 \div 1 = 1$. $1 \times 1 = 1$.

ART. 5. Cancel as much as possible in every instance, as it will afford every opportunity of

shortening the work, and in no case of lengthening it.

Explanation of Signs.

The sign	$=$	equality,	thus,	100 cents $=$ \$1
" "	$+$	addition,	"	$3+3=6$
" "	$-$	subtraction,	"	$6-3=3$
" "	$\times$	multiplication,	"	$3\times3=9$
" "	$\div$	division,	"	$9\div3=3$
			or thus,	$\frac{9}{3}=3$
			"	$3\|9=3$
" "	$:\,::\,:$	proportion, Read 2 is to 4, as 3 is to 6.	"	$2:4::3:6$
" "	$\surd$	square root, or radical, showing that the square root is required,	thus,	$\sqrt{9}=3$
" "	——	vinculum, showing that all under it is taken as one number,	thus,	$\sqrt{\overline{100-19}}=9$

The exponent (^{2}), shows that the number over which it is placed must be squared, thus, $6^2=36$

CANCELLATION.

Art. 6. First draw a perpendicular line, which in all instances separates the dividend from the divisor, or the factors of the dividend from those

of the divisor. The dividend, or its factors, is always placed on the right; the divisor, or its factors, on the left.

Art. 7. After having placed the sum on the line, divide the continued product on the right, by the continued product on the left.

Art. 8. Much mechanical labor may be saved,

1st, By canceling all equal numbers of noughts on opposite sides of the line.

2d, By canceling all equal numbers of figures.

3d, If the product of any numbers on one side will equal any number, or the product of any numbers, on the opposite side, by canceling all such numbers.

4th, By canceling all numbers that are divisible one by the other, * placing the quotient on the side of the greater.

5th, By continuing to cancel any numbers, (on opposite sides of the line), that are divisible by any supposed number, using the quotients.

6th, The operation may be completed by (Article 7).

Note. The answer always comes on the right. If the divisor is greater than the dividend, the answer will be a fraction.

N. B. The preceding should be committed to memory, and applied throughout the work.

* The divisions on the line must all be made without a remainder, until the last.

VULGAR FRACTIONS.

Art. 9. A fraction is a part of a unit. The bottom is the divisor, or denominator, showing into how many parts a unit is divided. The top is the dividend, or numerator, showing how many of such parts are taken.

Examples of the different kinds of Fractions.

$\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, etc., Proper fractions.

$\frac{2}{1}$, $\frac{3}{2}$, $\frac{4}{3}$, etc., Improper fractions.

$\left.\frac{\frac{3}{4}}{\frac{2}{3}},\ \frac{\frac{4}{5}}{\frac{4}{6}},\ \right\}$ Complex fractions.

$12\frac{1}{2}$, $6\frac{2}{3}$, etc., Mixed numbers.

Art. 10. 1. Reduce $6\frac{2}{3}$ to an improper fraction.

$$\frac{6\times3+2=20, \text{ numerator,}}{3, \text{ denominator.}}$$

2. Reduce $12\frac{1}{2}$ to an improper fraction. *Ans.* $\frac{25}{2}$.

Art. 11. 1. Reduce $\frac{75}{4}$ to a mixed number.

$$75\div4=18\frac{3}{4}.$$

2. Reduce $\frac{22}{3}$ to a mixed number. *Ans.* $7\frac{1}{3}$.

Art. 12. 1. Required to reduce $\frac{60}{120}$ to its lowest terms.

(See Article 8.) $\frac{\cancel{120}}{2}\Big|\frac{\cancel{60}}{1}$ *Ans.*

2. Reduce $\frac{90}{270}$ to its lowest terms. *Ans.* $\frac{1}{3}$.

Art. 13. To find the least common denominator of fractions.

Place to the left all the prime numbers of the denominators and the least number of prime factors that with the prime numbers, any two or more of them multiplied together will equal any of the composite denominators, their continued product will be the least common denominator.

1. Required to find the least common denominator of $\frac{1}{2}$, $\frac{2}{3}$, $\frac{1}{4}$, $\frac{1}{6}$, $\frac{3}{8}$.

$$\text{Divisors.}\left\{\begin{array}{r|l} 2 : 2 & 1=1 \\ 3 : 3 & 2=1 \\ 4 : 4 & 1=1 \\ : 6 & 1=1 \\ : 8 & 3=1 \end{array}\right\}\text{Quotients.}$$

Com. denom. 24

2. Reduce $\frac{3}{4}$, $\frac{7}{8}$, $\frac{5}{6}$, $\frac{4}{5}$, $\frac{9}{10}$, to a common denominator. *Ans.* 120.

Art. 14. Addition of Vulgar Fractions.

Rule. Find the least common denominator. For the numerators divide the common denominator by each particular denominator, and multiply the quotient by its numerator.

1. Add $\frac{5}{6}$, $\frac{1}{2}$, $\frac{9}{10}$, $\frac{11}{12}$, and $\frac{2}{5}$, together.

$$\left.\begin{array}{r|r|r} 6 : 6 & 5= 1 & 50 \\ 2 : 2 & 1= 1 & 30 \\ 5 : 10 & 9= 1 & 54 \\ : 12 & 11= 1 & 55 \\ : 5 & 2= 1 & 24 \end{array}\right\}\text{new numerators.}$$

$60|213=3\frac{11}{20}$ *Ans.*

2. Add $\frac{3}{4}$, $\frac{1}{2}$, $\frac{5}{6}$, $\frac{2}{3}$ and $\frac{7}{8}$ together. *Ans.* $3\frac{15}{24}$.

ART. 15. Subtraction of Vulgar Fractions.

Rule. Prepare the fraction as in Addition, and take the difference of the numerators.

1. From $\frac{3}{4}$, take $\frac{5}{12}$.

$$\begin{array}{r|r|l} 4:\ 4 & 3 = \ 1 & 9 \\ :12 & 5 = \ 3 & 5 \\ \hline & 12 & 4 = \frac{1}{3}\ \textit{Ans.} \end{array}$$

2. From $\frac{6}{7}$, take $\frac{5}{14}$. *Ans.* $\frac{1}{2}$.

ART. 16. Multiplication of Vulgar Fractions.

Rule. Place all the numerators on the right, and the denominators opposite.

1. Multiply $\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{4}{5}$ of $\frac{5}{6}$ of $\frac{6}{7}$, by 14.

$$\begin{array}{r|l} \cancel{2} & \cancel{1} \\ \cancel{3} & \cancel{2} \\ \cancel{4} & \cancel{3} \\ \cancel{5} & \cancel{4} \\ \cancel{6} & \cancel{5} \\ \cancel{7} & \cancel{6} \\ & \cancel{14}\ \ 2 \\ \hline & 2\ \ \textit{Ans.} \end{array}$$

2. Multiply $\frac{4}{5}$ of $\frac{8}{10}$ of $\frac{10}{16}$ of $\frac{3}{4}$, by $\frac{3}{12}$ of $\frac{5}{9}$ of 12. *Ans.* $\frac{1}{2}$.

Note. Of, between fractions always denotes multiplication.

ART. 17. Division of Vulgar Fractions.

Rule. Place the numerators of the dividend on the right, and the numerators of the divisor on the left. (See Art. 1.)

1. Divide $\frac{1}{2}$ of $\frac{5}{8}$ of $\frac{8}{21}$ of $\frac{14}{20}$, by $\frac{7}{12}$ of $\frac{1}{4}$ of $\frac{3}{21}$.

$$
\begin{array}{r|l}
\cancel{2} & 1 \\
\cancel{8} & \cancel{5} \\
\cancel{21} & \cancel{8} \\
\cancel{20} & \cancel{14} \\
\cancel{7} & \cancel{12} \\
1 & 4 \\
\cancel{3} & \cancel{21} \\
\hline
 & 4 \textit{ Ans.}
\end{array}
$$

2. Divide $\frac{3}{4}$ of $\frac{11}{24}$ of $\frac{8}{9}$ of $\frac{18}{22}$, by $\frac{1}{4}$ of $\frac{3}{12}$. *Ans.* 4.

Note. To understand multiplication and division of fractions, it is sufficient to observe Art. 9, 6 and 8.

ART. 18. Reduction of Complex Fractions.

Rule. Consider the numerator the dividend and the denominator the divisor, and proceed as in Division of Fractions.

1. $\dfrac{\frac{3}{4}}{\frac{2}{3}}$ numerator. denominator. This is the same as $\frac{3}{4} \div \frac{2}{3}$.

2. Reduce $\dfrac{\frac{4}{6}}{\frac{1}{3}}$ of $\dfrac{\frac{5}{8}}{\frac{10}{12}}$ of $\dfrac{30\frac{1}{4}}{5\frac{1}{2}}$ to a mixed number.

$$
\begin{array}{r|l}
\cancel{6} & \cancel{4} \\
1 & 3 \\
\cancel{8} & \cancel{5} \\
\cancel{10} & \cancel{12} \\
4 & \cancel{121}\ 11 \\
\cancel{11} & \cancel{2} \\
\hline
4 & 33 \\
\hline
 & 8\frac{1}{4} \textit{ Ans.}
\end{array}
$$

3. Reduce $\frac{\frac{5}{6}}{\frac{10}{18}}$ of $\frac{\frac{3}{4}}{\frac{6}{8}}$ of $\frac{8\frac{1}{4}}{5\frac{1}{2}}$. *Ans.* $2\frac{1}{4}$.

DECIMAL FRACTIONS.

ART. 19. Decimal Fractions are managed like whole numbers; and as their denominators are always 10, 100, etc., as $\frac{2}{10}$, $\frac{25}{100}$, etc., we express them by placing a decimal point to the left of the numerator; thus, .2, .25, etc. Noughts to the left decrease their value in a tenfold ratio.

Numeration Table.

Billions.	Hundreds of Millions.	Tens of Millions.	Millions.	Hundreds of Thousands.	Tens of Thousands.	Thousands.	Hundreds.	Tens.	Units.	Decimal Point.	Tenths.	Hundredths.	Thousandths.	Ten Thousandths.	Hundred Thousandths.	Millionths.	Ten Millionths.	Hundred Millionths.	Billionths.
4	4	4	4	4	4	4	4	4	4	.	4	4	4	4	4	4	4	4	4

Ascending. *Descending.*

Note. This table shows that the value of figures is determined by their distance from the decimal point.

Art. 20. Addition and subtraction of Decimals.

Rule. Place the decimal points directly under each other, and add, or subtract, as in whole numbers.

Add .4, .06, 1.12, 10.002, together.

$$\begin{array}{r} .4 \\ .06 \\ 1.12 \\ 10.002 \\ \hline 11.582 \end{array}\ \textit{Ans.}$$

From 3.856, take 2.412.

$$\begin{array}{r} 3.856 \\ 2.412 \\ \hline 1.444 \end{array}\ \textit{Ans.}$$

Art. 21. Multiplication of Decimals.

Rule. Multiply as in whole numbers, and point off in the product as many figures for decimals as there are decimal places in both factors.

Note. Point off from the right, and if there are not enough figures in the product to supply the decimal place, prefix noughts.

1. Multiply 2.34 by .12.

$$\begin{array}{r} 2.34 \\ .12 \\ \hline .2808 \end{array}$$

2. Multiply .275 by .25. *Ans.* .06875.

Art. 22. Division of Decimals.

Rule. Make an equal number of decimal places in both factors, by annexing ciphers to either, and divide as in whole numbers.

Note. The answer will be in whole numbers. If decimals are required, annex ciphers to the remainder, and continue the division.

Divide 2.3421 by 21.1.

$$21.1000\,|\,\underline{2.3421}\quad 0.111\ \textit{Ans.}$$

ART. 23. 1. What decimal is equivalent to $\frac{3}{4}$?

$$4\,|\,\underline{3.00}\quad .75\ \textit{Ans.}$$

2. What decimal is equivalent to $\frac{1}{8}$? *Ans.* .125.

ART. 24. 1. Reduce .75 to a vulgar fraction.

$$4\ \cancel{1}.\cancel{0}\cancel{0}\,|\,\underline{\cancel{7}\cancel{5}\ 3}\quad \tfrac{3}{4}\ \textit{Ans.}$$

2. Reduce .125 to a vulgar fraction. ***Ans.*** $\frac{1}{8}$.

FACTOR TABLES.

Federal Money.

10 mills (**m.**)	make 1 cent.	marked	*ct.*
100 cents	" 1 dollar.	"	**$**

Avoirdupois Weight.

16 drams (*dr.*)	make 1 ounce,	marked	*oz.*
16 ounces	" 1 pound,	"	*lb.*
25 pounds	" 1 quarter,	"	*qr.*
4 quarters	" 1 hundred weight,	"	*cwt.*
20 hundred	" 1 ton,	"	*T.*

Long Measure.

12 inches (*in.*)	make 1 foot,	marked	*ft.*
3 feet	" 1 yard,	"	*yd.*
5½ yards	" 1 pole or perch,	"	*p.*
40 poles	" 1 furlong,	"	*fur.*
8 furlongs	" 1 mile,	"	*M.*

Land, or Square Measure.

144 square inches(*sq.in*)	make 1 square foot,	marked	*sq. ft.*
9 square feet	" 1 square yard,	"	*sq. yd.*
30¼ square yards	" 1 square pole,	"	*P.*
40 square poles	" 1 rood,	"	*R.*
4 roods	" 1 acre,	"	*A.*

Solid, or Cubic Measure.

1728 solid inches (*s. in*)	make 1 solid foot,	marked	*s. ft.*
27 solid feet,	" 1 solid yard,	"	*s. yd.*
128 solid feet, 8X4X4	" 1 cord of wood,	"	*C.*
1¼ solid feet,	" 1 bushel,	"	*bu.*

NOTE. 1¼ solid feet make 1 bushel shelled grain, two measures are given when corn in the ear, (2½ solid feet.)

5 bushels make 1 barrel.

Time.

60 seconds (*sec.*)	make 1 minute,	marked	*min.*
60 minutes	" 1 hour,	"	*h.*
24 hours	" 1 day,	"	*d.*
30 days	" 1 month,	"	*m.*
12 months	" 1 year,	"	*y.*

NOTE. The limits of this work will not admit of superfluous matter, hence we have only used the factors that are necessary in *this* work, as others may be found in every work extant. In all practica work, 30 days are considered a month, and we have given the above table as a reference for practical purposes.

CAUSE AND EFFECT.

Art. 25. The older method of stating questions by Simple and Compound Proportion, and other rules, is very good as a mechanical contrivance; but the following is preferable, because of its greater simplicity and more extended applicability. It is an axiom in philosophy, that equal *causes* produce equal *effects*, and that *effects* are always proportionate to their *causes*. This principle gives rise to a rule applicable to all questions that can arise under proportion.

Rule. Any given cause is to its effect, as any required cause * *is to its effect; or, as another given cause to its required effect. Or, as any cause is to another similar cause, so is the effect of the first cause to the effect of the second.*

Previous to giving examples illustrative of the rule, we will give the definitions of a few terms as used in this work; and then merely mention the heads of rules as denominated in previous works, treating of their principles exclusively under *cause* and *effect*.

Simple Proportion.

Art. 26. 1. The *quotient* of one number divided by another is called the *ratio*. 2. Two

* Causes in the same proportion must always be of the same kind, and must be reduced to the same denomination before placed on the line.

numbers compared are called a *couplet*. 3. The first and second terms of a proportion constitute the first couplet. 4. The third and fourth terms constitute the second couplet. 5. The *first* and *fourth* terms are called the *extremes*. 6. The *second* and *third* terms, the *means*. 7. A *proportion* is an equality of *ratios*.

Art. 27. And since each *effect*, divided by its *cause*, or each *cause*, divided by its *effect*, produces equal *ratios*, it follows that the product of the *means* must equal the product of the *extremes*. This is true in every proportion, otherwise it is not a proportion.

Take the proportion — 2 : 4 :: 3 : 6. $2\times6=4\times3$.

Art. 28. The terms of any proportion may be changed *eight* different ways, and still constitute a proportion ; but the ratios will be different. Again, we may multiply or divide two proportions, term by term, and the result will be a proportion, etc. If we treat both couplets exactly alike, no matter what we do, the result will be a proportion, and the product of the *means* will equal the product of the *extremes*.

Art. 29. By retaining strong hold on this fact, we may find any lost term or factor of a term.

Take the proportion—2 : 4 :: 5 : 10.

Let the fourth term be wanting, or, as we shall denominate it, *blank*, as 2 : 4 :: 5 : [ ].*

* We shall use brackets to denote the place of the lost, or required term, or factor.

We still have the *means*, and one factor of the *extremes*. The means are complete, and the extremes incomplete; hence, in placing them on the line proceed thus:

Place the complete on the right, and the incomplete on the left; or, make a dividend of the complete, and a divisor of the incomplete.

Complete the operation by Art. 6, 7 and 8.

Note. In performing operations on the slate or blackboard, instead of placing the quotients to the right and left of their individual dividends, it will be more convenient to place them directly under them.

$$\begin{array}{r|l} \cancel{2} & \cancel{4}\ \ 2 \\ & 5 \\ \cline{2-2} & 10\ \textit{Ans} \end{array}$$

Let the second term be blank — 2 : [] :: 5 : 10

$$\begin{array}{r|l} \cancel{5} & \cancel{10}\ \ 2 \\ & 2 \\ \cline{2-2} & 4\ \textit{Ans.} \end{array}$$

Let the third term be blank — 2 : 4 :: [] : 10.

$$\begin{array}{r|l} \cancel{2}\ \cancel{4} & \cancel{10}\ \ 5 \\ & \cancel{2} \\ \cline{2-2} & 5\ \textit{Ans.} \end{array}$$

Let the first term be blank — [] : 4 :: 5 : 10.

$$\begin{array}{r|l} \cancel{2}\ \ \cancel{10} & \cancel{4}\ \ 2 \\ & \cancel{5} \\ \cline{2-2} & 2\ \textit{Ans.} \end{array}$$

ART. 30. The first caution is to strictly examine numbers, in doing which it will be seen that

in *cause* and *effect*, the actual numbers given need not be used, provided we use their proportionate numbers.

If 48 lbs. of pork cost 144 cts., what will 115 lbs. cost?

Cause.	*Effect.*	*Cause.*	*Effect.*
48 :	144 ::	115 :	[.]
1 :	3 ::	115 :	[ ].

This is simply another form of Art. 8. It is readily seen that 1 : 3, is as 48 : 144.

Compound Proportion.

ART. 31. In this system of statement, the philosophical idea is the only sure guide; hence, in stating a question, the mind should rest on denominations only; but after it is stated, we should look on the terms as abstract numbers.

N. B. When a correct statement is made, there will be the same number of elements, or factors, under the same letters, or in similar terms: as in the following example. If 2 men in 4 days can mow 5 acres of grass by working 10 hours per day, how many acres will 6 men mow in 2 days by working 12 hours per day?
Ans. 9 *A.*

Cause.	*Effect.*	*Cause.*	*Effect.*
Men, 2 :	*Acres,* 5 ::	*Men,* 6 :	Acres []
Days, 4 :		*Days,* 2 :	
Hours, 10 :		Hours, 12 :	

$$\begin{array}{r|l} \cancel{2} & \cancel{2} \\ \cancel{4} & \cancel{12}\ \ 3 \\ \cancel{2}\ \cancel{10} & \cancel{5} \\ & \cancel{6}\ \ 3 \\ \hline & 9\ \textit{Ans.} \end{array}$$

ART. 32. There are many questions that appear to be in *simple* proportion, that are really in *compound* proportion; the reason is, because of one term in each couplet being the same.

Example—If 4 men build a wall in 8 days, how long will it require 6 men to build it?

$$\begin{array}{cccc} \textit{Cause.} & \textit{Effect.} & \textit{Cause.} & \textit{Effect.} \\ 4 & :\ 1\ :: & 6 & :\ 1 \\ 8 & : & [\,] & \end{array} \qquad \begin{array}{r|l} 3\ \cancel{6} & \cancel{4}\ \ 2 \\ & 8 \\ \hline 3 & 16 = 5\tfrac{1}{3}\ \textit{Ans.} \end{array}$$

Note. Here one term in each couplet is one wall.

ART. 33. Similar questions sometimes give rise to the apparent view of more requiring less. Money must be compounded with time before it can produce interest, as more money will require less time to produce the same interest; but this is all apparent, for there is no such thing as more *cause* producing less *effect*.

ART. 34. This difficulty arises from not being able to readily determine which is *cause*, and which is *effect;* but this (the *only* difficulty to be met with in this system of statement), is readily overcome, when we consider that all action of any nature must be *cause*, and that which is accomplished, or follows such action, must be *effect*.

If 5 horses in 8 days consume 80 bushels of oats, how many bushels will 3 horses consume in 15 days?

$$\begin{array}{r|l} \cancel{5} & \cancel{80}\ 10 \\ \cancel{8} & 3 \\ & \cancel{15}\ 3 \\ \hline & 90\ \textit{Ans.} \end{array}$$

Cause.	*Effect.*	*Cause.*	*Effect.*
5 :	80 ::	3 :	[]
8 :		15 :	

Note. Here it is evident that the action of the horses, multiplied by the days, in both couplets, must express the *cause*, and the consumption of the oats is the *effect*.

ART. 35. There are many questions, however, where it is indifferent which is taken for cause, and which for effect; only, observe when one thing is taken for cause in the first couplet, a similar one must be taken for cause in the second couplet.

If the transportation of 4 cwt. 12 miles cost $10, how far may 6 cwt. be carried for $15. *Ans.* 12 *M.*

$$\begin{array}{r|l} \cancel{10} & \cancel{4} \\ \cancel{6} & 12 \\ & \cancel{15} \\ \hline & 12\ \textit{A.} \end{array}$$

Cause.	*Effect.*	*Cause.*	*Effect.*
10 :	4 ::	15 :	6.
	12 ::		[].

Or thus:

Cause.	*Effect.*	*Cause.*	*Effect.*
4 :	10 ::	6 :	15.
12 :		[] :	

Or thus:

Cause.	*Cause.*	*Effect.*	*Effect.*
4 :	6 ::	10 :	15.
12 :	[] ::		

Or thus:

Effect.		*Effect.*		*Cause.*		*Cause.*
15	:	10	::	6	:	4.
				[]	:	12.

The same terms, multiplied together in each of the different statements, show that this method is strictly scientific.

1. If the wages of 6 men, 14 days, be $84, what will be the wages of 9 men for 16 days? *Ans.* $144.

2. If I lend $400 to a friend for 16 months, how long ought he to lend me $1600 to return the favor? (See Art. 32.) *Ans.* 4 *mos.*

3. If $2\frac{1}{2}$ yds. of cloth, $2\frac{2}{5}$ yds. wide, cost $3.35, how many yds., that is $1\frac{1}{2}$ yds. wide, can I have for $134.00? *Ans.* 160 *yds.*

4. If 11 men in 7 days, working 13 hours per day, dig a ditch that is $37\frac{3}{4}$ ft. long, $2\frac{2}{5}$ ft. wide, $3\frac{1}{2}$ ft. deep, in how many days can $5\frac{1}{2}$ men, working 14 hours per day, dig another that is $18\frac{7}{8}$ ft. long, $14\frac{4}{5}$ ft. wide, and $10\frac{1}{2}$ ft. deep? *Ans.* $120\frac{1}{4}$ *days.*

Note. In stating all questions in Cause and Effect, see Art. 25.

Practice.

The preceding principles of *Cause* and *Effect* will apply to the solution of all questions arising in Practice, which is nothing more than Simple Proportion, having 1 for the first term.

Interest.

We omit the definitions of terms used in

treating of Interest, they being so common that the learner is supposed to be familiar with them.

Art. 36. The system of *Cause and Effect* is very extensive and easy in its application. It covers every case that can arise under Interest. To find the interest, principal, time, or rate per cent., and thus dispense with five or six special rules, as found in almost every Arithmetic, we will use the following

Interest Table.

Cause.	*Effect.*		*Cause.*	*Effect.*
100 :	Rate pr. ct.	: :	Principal :	Interest.
1 year :*			Time :	

☞ *Make the blank where the table designates the term, or factor, you wish to find.*

What is the interest of $200, for 3 years, at 6 per cent. ?

Cause.	*Effect.*		*Cause.*	*Effect.*
100 :	6	: :	200 :	[]
1 :			3 :	

$\cancel{100}$	$2\cancel{00}$
1	3
	6
	$36 *Ans.*

Art. 37. What principal at interest for 24 months, at 6 per cent., will gain $48 ?

Cause.	*Effect.*		*Cause.*	*Effect.*
100 :	6	: :	[] :	48
12 :			24 :	

$\cancel{6}$	$\cancel{48}$ 2
$\cancel{24}$	100
	$\cancel{12}$ 2
	$400 *Ans.*

* Or the factors of a year. If the time be months, 12 ; if days, 12 and 30 ; if weeks, 52, etc

Art. 38. At what rate per cent. will $200, in 450 days, gain $15 interest?

Cause.	*Effect.*		*Cause*	*Effect.*
100 :	[]	::	200 :	15
12 :			450 :	
30 :				

$\cancel{200}$	$\cancel{100}$	
$\cancel{450}$	$\cancel{12}$	6
	$\cancel{30}$	
	$\cancel{15}$	
	6 pr. ct. *Ans.*	

Art. 39. In what time will $160 gain $2 interest at 3 per cent?

Cause.	*Effect.*		*Cause.*	*Effect.*
100 :	3	::	160 :	2
12 :			[] :	

\$ $\cancel{160}$	$\cancel{100}$	5
$\cancel{3}$	$\cancel{12}$	
	$\cancel{2}$	
	5 mos. *Ans.*	

Note. Many abridged rules might be given for the solution of interest questions; we shall, however, give but few, as we are satisfied that those who make themselves acquainted with the preceding general principles, will be able to make their own abridgments.

Art. 40. 1. When the time is months, and rate per cent. 6, to find the interest, multiply the principal by half the number of months.

2. When days, divide them by 60, and multiply the quotient by the principal.

3. When the time is months, and the rate per cent. 4, multiply the principal by $\frac{1}{3}$ the number of months.

4. When the time is months, and the rate per cent. 3, multiply the principal by $\frac{1}{4}$ the number of months.

5. To find the interest of any principal, for any time, at any rate per cent:

Make a dividend of the principal, time, and rate per cent. If the time be months, the divisor is 12; *if days,* 12 *and* 30, *etc.*

Note. When the time is given in different denominations — as months, days, etc. — it must first be reduced to the lowest denomination mentioned, then placed on the line.

Miscellaneous Examples.

1. What is the interest of $100, for 3 years, at 5 per cent.? *Ans.* $15.00.
2. What is the interest of $21, for 1 year and 4 months, at 3 per cent.? *Ans.* 84 cts.
3. What is the interest of $50, for 18 months, at 4 per cent.? *Ans.* $3.00.
4. What is the interest of $84, for 6 months and 20 days, at 9 per cent.? *Ans.* $4.20.
5. What is the interest of $75, for 60 days, at 8 per cent.? *Ans.* $1.00.
6. What is the interest of $46, for 90 days, at 6 per cent.? *Ans.* 69 cts.

Note. To save space in giving other problems, use the interest in the preceding, and find the principal, time, and rate per cent., alternately, by the Interest Table (Art. 36).

Percentage.

Art. 41. To find the amount for which an article must be sold, to gain or lose any given rate per cent.

State thus: As 100 *is to* 100 *with the gain per cent. added, or loss per cent. subtracted, so is the prime cost to the required price.*

1. A merchant paid 44 cents per yard for cloth, for what must he sell it to gain 25 per cent.?

Cause. *Effect.* *Cause.* *Effect.*
100 : 100+25 :: 44 : []

$$\begin{array}{r|l} 4\ \cancel{100} & \cancel{125}\ \ 5 \\ & \cancel{44}\ \ 11 \\ \hline & A.\ 55 \text{ cts.} \end{array}$$

2. Paid \$80 for a horse, for what must he be sold to gain 50 per cent.? *Ans.* \$120.

3. Paid \$90 for a horse, how must he be sold to lose $33\frac{1}{3}$ per cent.?

Cause. *Effect.* *Cause.* *Effect.*
100 : 100—$33\frac{1}{3}$ :: 90 : [].

$$\begin{array}{r|l} \cancel{3} & \cancel{90}\ \ 30 \\ \cancel{100} & \cancel{200} \\ \hline & \$60\ \textit{Ans.} \end{array}$$

4. Paid \$100 for sheep, how must they be sold to lose 20 per cent.? *Ans.* \$80.

Art. 42. Having the cost and selling price given to find the gain or loss per cent.

State thus: 100 *is to the required gain or loss per cent. as the prime cost to the difference between the prime cost and selling price.*

1. A merchant bought cloth at 48 cts. per yd., and sold it at 60 cts. what was the gain per cent.

$$\begin{array}{r|l} & \cancel{12} \\ 4\ \cancel{48} & \cancel{100}\ \ 25 \\ \hline & A.25 \text{ cts.} \end{array}$$

Cause. *Effect.* *Cause.* *Effect.*
100 : [] :: 48 : 60—48

2. Had he paid 60 cts., and sold it for 48 cts., what would have been the loss per cent.?

$$\begin{array}{r|l} \cancel{60} & \cancel{12}\ 2 \\ & 10\cancel{0} \\ \hline & A.20 \text{ cts.} \end{array}$$

Cause.	*Effect.*	*Cause.*	*Effect.*
100	: []	:: 60	: 60—48

3. A farmer paid $75 for a horse, and $150 for a chaise; he sold the horse for $100, and the chaise for $125. What per cent. did he gain on the horse, and lose on the chaise?

Ans. $\begin{cases} 33\frac{1}{3} \textit{ per cent. gained.} \\ 16\frac{2}{3} \textit{ per cent. lost,} \end{cases}$

ART. 43. Having the selling price of an article, and the rate per cent., gained or lost, given, to find the cost.

State thus: 100 *is to* 100 *with the gain per cent. added, or loss per cent. subtracted, as the required cost to the the selling price.*

1. Having sold a watch for $14, I thereby lost 30 per cent., what did it cost me?

$$\begin{array}{r|l} \cancel{70} & \cancel{14}\ \ 2 \\ & 10\cancel{0} \\ \hline & 20 \ \textit{Ans.} \end{array}$$

Cause.	*Effect.*	*Cause.*	*Effect.*
100 :	100—30	:: []	: 14.

2. If a farm be sold for $220, and 10 per ct. is gained, what did it cost?

$$\begin{array}{r|l} \cancel{110} & 100 \\ & \cancel{220}\ 2 \\ \hline & \$200 \ \textit{Ans.} \end{array}$$

Cause.	*Effect.*	*Cause*	*Effect.*
100 :	100+10 ::	[]	220.

3. I sold a horse for $40, and by so doing lost 20 per cent.; whereas, I ought, in trading, to have cleared 30 per cent. How much was he sold for under his real value?

Cause.	*Effect.*	*C.*	*E.*
100—20 :	100 ::	40 :	[ ]=50
100 :	100+30 ::	50 :	[ ]=65—40=$25 *Ans.*

Or thus:

$$\begin{array}{r|l} \cancel{2}\ \cancel{80}\ \cancel{100} & \\ \cancel{100} & \cancel{130}\ 65 \\ & \cancel{40} \\ \hline \end{array}$$

65—40=$25 *Ans.*

Barter.

ART. 44. (See Article 32). The term understood in Barter is one amount.

1. How many bushels of apples must I have, at 25 cts. pr. bu., for 35 yds. of calico, at 20 cts. per yard?

Cause.		*Effect.*		*Cause.*		*Effect.*
[ ]	:	1	::	35	:	1
25	:			20		

$$\begin{array}{r|l} 1 & 1 \\ \cancel{5}\ \cancel{25} & \cancel{35}\ 7 \\ & \cancel{20}\ 4 \\ \hline & 28\ \textit{Ans.} \end{array}$$

2. If I barter 12 bu. of flaxseed, at 65 cts. pr. bu., for cloth, at 60 cts. pr. yd. How many yards must I have?

Cause.		*Effect.*		*Cause.*		*Effect.*
12	:	1	::	[ ]	::	1
65	·			60	::	

$$\begin{array}{r|l} & \cancel{12} \\ \cancel{5}\ \cancel{60} & \cancel{65}\ 13 \\ \hline & 13\ \textit{Ans} \end{array}$$

Discount.

ART. 45. *State thus: As* 100 *is to the amount of* 100 *for the given time, at the given rate per cent., so is the required present worth, to the given debt.*

Note. Subtracting the present worth from the amount, will give the discount,

1, What is the discount of $436, for 18 months, at 6 per cent.?

Cause. Effect. Cause. Effect.
100 : 109 :: [] : 436.

$$\begin{array}{r|l} \cancel{109} & 100 \\ & \cancel{436}\ 4 \\ \hline & \textit{Pres. worth } \$400 \end{array}$$

436—400=36 *Ans.*

The following process is preferable for its brevity:

Make a dividend of the principal and time; multiply one year (or its equivalent in months or days) by 100; *divide by the rate per cent., and add the time (of the same denomination) to the quotient, for a divisor.*

N. B. The result will be the discount in the same denomination of the sum given.

To find the present worth, subtract the discount from the sum given.

Take the preceding example:

$12\times100\div6=200$. $200+18=218$, *divisor.*

$$\begin{array}{r|l} \cancel{218} & \cancel{436}\ 2 \\ & 18 \\ \hline & \$36 \textit{ discount.} \end{array}$$

To prove discount.

The interest of the present worth must equal the discount of the sum, for the same time, at the same rate per cent.

$\cancel{2}$ $\cancel{12}$	$\cancel{400}$ 2
$\cancel{100}$	18
	$\cancel{6}$
	\$36 *interest*=\$36 *discount.*

2. What is the discount of \$80, for 200 days, at 12 per cent.?

$360\times100\div12=3000.$ $3000+200=3200$ *div'r.*

$\cancel{16}$ $\cancel{3200}$	$\cancel{80}$ 5
	$\cancel{200}$
	\$5 *Ans.*

3. What is the discount of \$48, for 4 years, at 5 per cent.? *Ans.* \$8.

4. What is the discount of \$218, for 9 mos., at 12 per cent.? *Ans.* \$18.

5. What is the discount of \$1140, for 120 days, at 4 per cent.? *Ans.* \$15.

6. What is the difference between the interest of \$160, for 400 days, at 6 per cent, and the discount of the same sum, for the same time, at the same per cent.? *Ans.* $66\frac{2}{3}$ cts.

Measurement.

Art. 46. For all measurement, use the following table in stating — or rule, if *Cause* and *Effect* be read alternately :

Mensuration Table.

Cause.		*Effect.*		*Cause.*		*Effect.*
Factors of the unit of measure.	:	Unit of measure.	: :	Factors of the thing to be measured.	:	Number of units.*

☞ *Make the blank in the term where the table designates the term or factor sought.*

Cord Wood.

ART. 47. 1. How many cords are there in a pile of wood 80 feet long, 4 feet wide, 8 feet deep?

Cause.		*Effect.*		*Cause.*		*Effect.*
8	:	1	: :	80	:	[ ].
4	:			4	:	
4	:			8	:	

$\cancel{8}$	$\cancel{80}$ 20
$\cancel{4}$	$\cancel{4}$
$\cancel{4}$	$\cancel{8}$
	20 *Ans.*

2. How much will a pile of wood cost, that is 48 ft. long, $2\frac{1}{2}$ ft. wide, and 16 ft. deep, at $1 per cord? *Ans.* $15.00

3. How long must a pile of wood be to contain 13 cords, that is $6\frac{1}{2}$ ft. wide, and 4 ft. deep?

Cause.		*Effect.*		*Cause.*		*Effect.*
8	:	1	: :	[ ]	:	13.
4	:			$6\frac{1}{2}$	:	
4	:			4	:	

$\cancel{13}$	2
$\cancel{4}$	$\cancel{13}$
	8
	$\cancel{4}$
	4
	64 *Ans.*

* The number of units is the thing, or answer, sought; but if this be given some factor is sought.

4. How wide must a pile of wood be, that is 24 ft. long, and 16 ft. deep, to contain 12 cords ? *Ans*. 4 ft.

Land Measure.

Art. 48. 1. How many acres of land in a field 80 rods square ?

Cause.		*Effect.*		*Cause.*		*Effect.*
4	:	1	::	80	:	[].
40	:			80	:	

$\cancel{4}$	$\cancel{80}$ 20
$\cancel{40}$	$\cancel{80}$ 2
	40 *Ans.*

2. What is the area of a rectangular field 60 rods long, and 121 yards wide ?

Cause.		*Effect.*		*Cause.*		*Effect.*
4	:	1	::	60	:	[].
40	:			121	:	
$5\frac{1}{2}$	:					

4	$\cancel{60}$ 3
$\cancel{2}$ $\cancel{40}$	$\cancel{121}$ 11
$\cancel{11}$	$\cancel{2}$
4	33
	Ans. $8\frac{1}{4}$.

3. How wide must a rectangular lot be that is 24 rods long, to contain 3 acres ?

Cause.		*Effect.*		*Cause.*		*Effect.*
4	:	1	::	24	:	3.
40	:			[ ]	:	

$\cancel{3}$ $\cancel{24}$	$\cancel{40}$ 5
	4
	$\cancel{3}$
	20 *Ans.*

Note. This principle is applicable to the measurement of lands of all shapes, as given under *Mensuration.*

Carpeting.

ART. 49. 1. How many yards of carpeting, that is $\frac{3}{4}$ of a yard wide, will be required to cover a floor, 27 feet long, and 13 feet wide?

Cause.		*Effect.*		*Cause.*		*Effect.*
1	:	1	: :	27	:	[ ].
$\frac{3}{4}$	:			13	:	
9						

$$\begin{array}{r|l} \cancel{9} & \cancel{27} \\ \cancel{3} & 13 \\ & 4 \\ \hline & 52 \text{ yds. } A. \end{array}$$

Note. The 9 under the *first cause* is to reduce it to square feet. to be of the same denomination of the *second cause.*

2. How wide must a floor be, that is 18 feet long, to require 12 square yards to cover it?

Cause.		*Effect.*		*Cause.*		*Effect.*
1	:	1	: :	18	:	12
1	:			[]	:	
9						

$$\begin{array}{r|l} \cancel{2}\ \cancel{18} & \cancel{12}\ 6 \\ & \cancel{9} \\ \hline & 6 \text{ ft. } A. \end{array}$$

3. What will it cost to carpet a room that is 36 feet long, 10 feet wide — carpeting $1\frac{1}{4}$ yards wide, and worth $37\frac{1}{2}$ cents per yard?

Cause.		*Effect.*		*Cause.*		*Effect.*
1	:	1	: :	36	:	[ ].
$1\frac{1}{4}$	:			10	:	
9		$37\frac{1}{2}$			:	

$$\begin{array}{r|l} \cancel{9} & \cancel{36}\ 4 \\ \cancel{5} & \cancel{10} \\ \cancel{2} & 75 \\ & 4 \\ \hline & \$12.00 \ A. \end{array}$$

Crib and Box Measure.

ART. 50. 1. How many barrels of corn will a crib hold, that is 50 ft. long, 9 ft. wide, and **3** ft. **4** in. deep?

Cause.		Effect.		Cause.		Effect.
$2\frac{1}{2}$	:	1	::	50	:	[].
5				9	:	
				3ft4in	:	

$\cancel{5}$	$\cancel{50}$ 2
$\cancel{5}$	2
$\cancel{3}$	$\cancel{9}$ 3
	10
	120 bbls. *A.*

2. How many bushels of corn will a crib hold, that is 18 ft. 9 in. long, 8 ft. high, 7 ft. 6 in. wide?

Cause.		Effect.		Cause.		Effect.
$2\frac{1}{2}$	:	1	::	8	:	[]
				18ft9in	:	
				7ft6in	:	

$\cancel{5}$	2
$\cancel{12}$	225
$\cancel{12}$	$\cancel{90}$
	$\cancel{8}$
	450 bu. *A.*

3. How high must a crib be, to contain 360 bushels of corn, when it is 18 feet long, and 6 feet 3 inches wide?

Cause.		Effect.		Cause.		Effect.
$2\frac{1}{2}$	:	1	::	18	:	360
				6ft3in		
				[]	:	

$\cancel{2}$	$\cancel{5}$
$\cancel{18}$	$\cancel{360}$ $\cancel{20}$ 4
$\cancel{5}$ $\cancel{25}$	$\cancel{4}$ 2
	8 ft. *Ans*

4. How many bushels of wheat in a box, 10 feet long, 3 feet wide, and 2 feet 8 inches high?

Cause.		Effect.		Cause.		Effect.
$1\frac{1}{4}$	:	1	:	10	:	[].
				3	:	
				2ft8in		

$\cancel{5}$	$\cancel{4}$
	$\cancel{10}$ 2
	$\cancel{3}$
$\cancel{12}$	32
	64 *Ans.*

5. How long must a box be, that is 4 feet 2 inches wide, and 2 feet 1 inch deep, to contain 50 bushels?

Cause.		*Effect.*		*Cause.*		*Effect.*
$1\frac{1}{4}$	:	1	: :	4 ft. 2 in.	:	50
				2 ft. 1 in.	:	
				[]		

$\cancel{4}$	5
$\cancel{50}$	$\cancel{50}$
5 $\cancel{25}$	$\cancel{12}$ 3
	12
5	36
	$7\frac{1}{5}$ ft. *A.*

6. How many panes of glass in a box of 50 feet, 8 inches by 10 inches?

Cause.		*Effect.*		*Cause.*		*Effect.*
8	:	1	: :	50	:	[]
10	:			12		
				12		

$\cancel{2}$ $\cancel{8}$	$\cancel{12}$ 3
$\cancel{10}$	$\cancel{12}$ 6
	$\cancel{50}$
	90 *A.*

7. How many feet of glass in a box, that contains 120 panes, 10 in. wide, and 12 in. long?

Cause.		*Effect.*		*Cause.*		*Effect.*
10	:	1	: :	[]	:	120.
12	:			12		
				12		

$\cancel{12}$	$\cancel{120}$ 10
$\cancel{12}$	10
	$\cancel{12}$
	100 *Ans.*

Note. The 12s under the *second cause* in the preceding examples are to reduce them to square inches, to be of the same denomination of the *first cause.*

House Covering.

ART. 51. 1. How many shingles will be required to cover a house, that is 24 ft. long, and 15 ft. wide?

Cause.		Effect.		Cause.		Effect.
4	:	1	::	24	:	[].
6	:			15	:	
				$\frac{4}{3}$		
				$\frac{12}{12}$		

$\cancel{4}$	12
$\cancel{6}$	12
	$\cancel{24}$
	$\cancel{15}$ 5
$\cancel{3}$	4
	2880 *Ans.*

Note. It is customary to allow the shingles to be 4 inches wide, and to show 6 inches, and the rafters to be $\frac{2}{3}$ the width of the house, both making $\frac{4}{3}$. When this is the case, the process may be shortened by

Multiplying the product of the length and width of the house, by 8.

2. How many shingles will be required to cover a building, that is 30 feet long, and 25 feet wide? *Ans.* 6000.

Art. 52. 1. How many boards will be required to cover a house 36 ft. long, 24 ft. wide; the boards 6 in. wide, and to show 18 inches?

Cause.		Effect.		Cause.		Effect.
6	:	1	::	36	:	[]
18	;			24	:	
				$\frac{4}{3}$	:	
				$\frac{12}{12}$		

$\cancel{6}$	$\cancel{12}$ 2
$\cancel{18}$	$\cancel{12}$ 4
	24
	$\cancel{36}$ 2
$\cancel{3}$	4
	1536 *Ans.*

2. How much must I pay for boards 6 inches wide, and to show 15 inches, at $6 per 1000, to cover a house 24 feet long, and 20 feet wide?

Cause.	Effect.	Cause.	Effect.
6 :	1 ::	24 :	[ ].
15 :		20 :	
1000 :	6 ::	$\frac{4}{3}$:	
		12	
		12	

$\cancel{6}$	$\cancel{12}$ $\cancel{4}$
5 $\cancel{15}$	$\cancel{12}$ 4
25 $\cancel{1000}$	24
	$2\cancel{0}$
$\cancel{3}$	4
	$\cancel{6}$
125	768
	\$6 14c. 4m

Brick Work.

Art. 53. 1. How many bricks, 8 inches long, 4 inches wide, will be required to pave a walk 3 feet 4 inches wide, and $\frac{1}{6}$ of a mile long?

Cause.	Effect.	Cause.	Effect.
8 :	1 ::	$\frac{1}{6}$:	[ ].
4		3ft4in :	
		12	
		12	
		3	
		$5\frac{1}{2}$	
		40	
		8	

$\cancel{8}$	$\cancel{12}$
$\cancel{4}$	$\cancel{12}$
$\cancel{12}$	$\cancel{40}$ 10
$\cancel{6}$	1
	3
$\cancel{2}$	11
	40
	$\cancel{8}$
	13200 *A.*

2. How many bricks will be required to build the walls of a house 20 feet long, 15 feet wide, 16 feet high, and 8 inches thick, allowing $\frac{1}{6}$ for mortar, the brick to be 8 inches long, 4 inches wide, and $2\frac{1}{2}$ inches thick?

Cause.		*Effect.*		*Cause.*		*Effect.*
8	.	1	::	35	:	[ ].
4	:			2	:	
$2\frac{1}{2}$	:			16	:	
				8	:	
				$\frac{5}{6}$	:	
				$\frac{12}{12}$		

$\cancel{8}$	12
$\cancel{4}$	$\cancel{12}$
$\cancel{5}$	$\cancel{2}$
$\cancel{6}$	35
	2
	16
	$\cancel{8}$
	$\cancel{5}$
	13440 *A.*

Note. Adding the length and width together, and doubling the sum, gives one straight wall. Multiply that by the height and thickness and by $\frac{5}{6}$, (making a deduction of $\frac{1}{6}$ for mortar).

N. B. Deductions must be made for windows and doors.

Log and Plank Measure.

Art. 54. 1. How many solid feet in a log, 21 inches in diameter, and 16 feet long?

Cause.		*Effect.*		*Cause.*		*Effect.*
12	:	1	::	21	:	[ ].
12	:			21	:	
12				$\frac{11}{14}$	:	
				16	:	
				12		

	$\cancel{12}$	$\cancel{12}$	
$\cancel{4}$	$\cancel{12}$	$\cancel{21}$	7
$\cancel{3}$	$\cancel{12}$	$\cancel{21}$	$\cancel{3}$
2	$\cancel{14}$	11	
		$\cancel{16}$	$\cancel{4}$
2		77	
		$38\frac{1}{2}$ *A.*	

Note. To find the solid contents of a log, we first find the area of the end by Art. 68, and multiply by the length.

2. How many square feet of plank, for ceiling, flooring, etc. (1 inch thick), in a log, 24 inches in diameter, 20 feet long, allowing $\frac{1}{4}$ for saw-calf ?

Cause.		*Effect.*		*Cause.*		*Effect.*
12	:	1	::	24	:	[ ].
12	:			24	:	
$1\frac{1}{4}$				20	:	
2				12		

$\cancel{12}$	$\cancel{24}$
$\cancel{2}$	24
$\cancel{5}$	4
	$\cancel{20}$ 4
$\cancel{12}$	$\cancel{12}$
	384 *Ans.*

Note. Dividing by 2, throws away the slabs in getting the area of the end. (See Art. 61.)

3. How many square feet of plank, 1 inch thick, in a log 30 inches in diameter and 12 feet long, allowing $\frac{1}{4}$ for saw-cut ? *Ans.* 360.

4. How many square feet of plank. 1 inch thick, in a log 48 inches in diameter, and 10 feet long, allowing $\frac{1}{4}$ for the saw-cut ? *Ans.* 768.

5. How many square feet of sheeting plank, $\frac{3}{4}$ of an inch thick (including saw-cut), in a log, 12 feet long, and 7 inches in diameter ?

Cause.		*Effect.*		*Cause*		*Effect.*
12	:	1	::	12	:	[ ]
$_{12}\frac{3}{4}$	:			7	:	
				7	:	
				$1\frac{1}{4}$:		
				12		

$\cancel{12}$	$\cancel{12}$
3	$\cancel{4}$ 2
$\cancel{2}$ $\cancel{14}$	7
$\cancel{12}$	7
	11
	$\cancel{12}$
3	154
	$51\frac{1}{3}$ *Ans.*

6. To cut $7\frac{1}{2}$ square feet off of a plank 6 inches wide, how many feet of its length must be taken ?

Cause.		Effect.		Cause.		Effect.
12	:	1	::	[]	:	$7\frac{1}{2}$.
1:				6	:	
				12		

$$\begin{array}{r|l} \cancel{12} & \cancel{12} \\ \cancel{6} & \cancel{12} \\ \cancel{2} & 15 \\ \hline & 15 \textit{ Ans.} \end{array}$$

Promiscuous Examples.

1. How many acres are there in a round field, 56 rods in diameter?

Cause.		Effect.		Cause.		Effect.
4	:	1	::	56	:	[].
40	:			56	:	
				$\frac{11}{14}$	:	

$$\begin{array}{r|l} \cancel{4} & \cancel{56}\ 7 \\ 5\ \cancel{40} & \cancel{56} \\ \cancel{14} & 11 \\ \hline 5 & 77 \\ \hline & 15\frac{2}{5}\ \textit{A.} \end{array}$$

2. If I send 12 bushels of wheat to mill, how many pounds of flour will I get, allowing $\frac{1}{12}$ for toll, $\frac{1}{8}$ for bran, $\frac{1}{6}$ for shorts; weight of wheat, 60 lbs. per bushel?

Cause.		Effect.		Cause.		Effect.
12	:	11	::	12	:	[].
8	:	7	::	60	:	
6	:	5	::		:	

$$\begin{array}{r|l} \cancel{12} & 11 \\ 4\ \cancel{8} & 7 \\ \cancel{6} & 5 \\ & \cancel{12} \\ & \cancel{60}\ \cancel{10}\ 5 \\ \hline 4 & 1925 \\ \hline & 481\frac{1}{4}\ \textit{A.} \end{array}$$

3. I wish to get $481\frac{1}{4}$ lbs. of flour; how many bushels of wheat must I send to mill, making the same allowances as in the preceding example?

$\cancel{12}$ $\cancel{60}$	$\cancel{1925}$ $\cancel{385}$
$\cancel{11}$	$\cancel{12}$
$\cancel{7}$	$\cancel{8}$ 2
$\cancel{5}$	6
$\cancel{4}$	
	12 *Ans.*

Cause.		*Effect.*		*Cause.*		*Effect.*
12	:	11	::	[ ]	:	$481\frac{1}{4}$
8	:	7	::	60	:	
6	:	5	::			

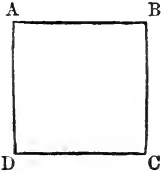
4. How many barrels of corn in a field 240 hills long, by 160 wide, each hill to average 2 ears, 120 of which will make 1 bushel?

Cause.		*Effect.*		*Cause.*		*Effect.*
120	:	1	::	160	:	[ ].
5	:		::	240	:	
				2	;	

$\cancel{120}$	$\cancel{240}$ 2
$\cancel{5}$	$\cancel{160}$ 32
	2
	A. 128

MENSURATION OF SUPERFICES.

ART. 55. To measure a square.

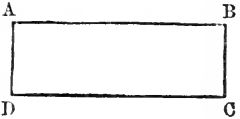


Rule. Multiply the side A B into A D.

Let A B=12. A D=12.
Then, $12 \times 12 = 144$.

Note. The area will be of the same denomination that the sides are.

Art. 56. To measure a rectangle.



Rule. Multiply the length by the width.

Let A B=40. A D=12.

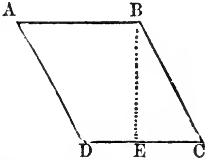
Then, $40\times12=480$.

Art. 57. When the area and one side are given, to find the other.

Rule. Divide the area (reduced to the same denomination as the side), by the given side.

In the last figure, A B=40, and area=480, to find A D. $480\div40=12$.

Art. 58. To measure a rhombus.

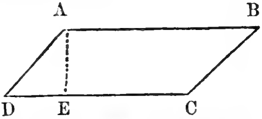


Rule. Multiply one side by the shortest distance between the sides.

Let A B=16. B E=12.

Then, $16\times12=192$.

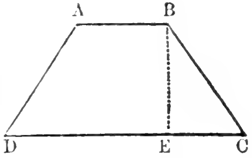
Art. 59. To measure a rhomboid.



Rule. Multiply one of the longer sides by the shortest distance between them.

Let A B$=40$. A E$=16$.
Then, $40\times16=640$.

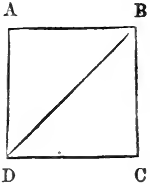
Art. 60. To measure a trapezoid.



Rule. Multiply the half sum of the parallel sides by the shortest distance between them.

Let A B$=8$ D C$=18$. B E$=10$.
Then, $18+8\div2\times10=130$.

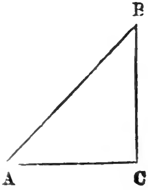
Art. 61. The diagonal of a square given to find the area.



Rule. Multiply the diagonal by half itself.

Let B D$=80$.
Then, $80\times40=3200$.

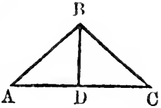
Art. 62. To measure a right-angled triangle.



Rule. Multiply the base by the perpendicular, and take half.

Let A C$=16$. B C$=19$.
$19\times16\div2=152$.

ART. 63. To measure an acute or obtuse-angled triangle.

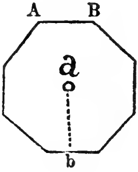


Rule. Multiply the base by a perpendicular line from the vertex to the base, and take half.

Let A C=60. B D=24. $60 \times 24 \div 2 = 720$.

Note. Take the longest side for the base.

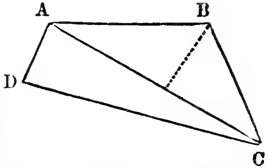
ART. 64. To measure any regular polygon.



Rule. Multiply one side by the perpendicular distance from the center; take half the product, and multiply the quotient by the number of sides.

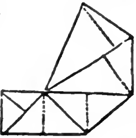
Let A B=15, a b=20, (No. of sides, 8).
$15 \times 20 \div 2 \times 8 = 1200$.

ART. 65. To measure a trapezium.



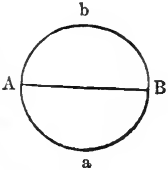
Rule. Draw a diagonal line, and calculate the two triangles by Art. 63, their sum will be the area of the trapezium.

Art. 66. To measure any irregular figure.



Rule. First cut it into triangles by drawing diagonal lines. Calculate (by Article 63) *the several triangles, and their sum will be the area of the figure.*

Art. 67. To measure a circle. Given, the diameter of a circle, to find the circumference.



Rule. Multiply the diameter by $\frac{22}{7}$.

Let A B$=14$. $14\times\frac{22}{7}=44$.

Note. When the circumference is given, to find the diameter, multiply the circumference by $\frac{7}{22}$, (the converse of the preceding.)

Let A a B b$=44$, to find A B. $44\times\frac{7}{22}=14$.

Art. 68. To find the area of a circle.

Rule. Multiply the circumference by the diameter, and take one-fourth.

In the preceding $44\times14\div4=154$,

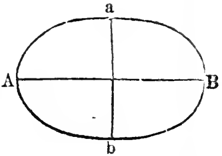
Or, multiply the square of the diameter by $\frac{11}{14}$.

Art. 69. To find the diameter of a circle equal to a square whose side is given.

Rule. Multiply the side by 1.128.

Let the side$=10$. Then, $10\times1.128=11.280$.

ART. 70. To find the area of an ellipse.



Rule. Multiply the product of the transverse and conjugate diameters (A B *and* a b) *by* $\frac{11}{14}$.

Let A B=14. a b=10.

14 | 14
10
11

ART. 71. To find the area of a globe.

Rule. Multiply the circumference by the diameter; or, multiply the square of the diameter by $\frac{22}{7}$.

ART. 72. To find the area of a cylinder, or round body of equal largeness from end to end.

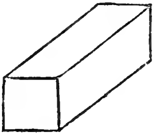
Rule. Multiply the circumference by the length.

ART. 73. To find the area of a right cone.

Rule. Multiply the circumference of the base by the slant height, and take half.

MENSURATION OF SOLIDS.

ART. 74. To find the solidity of a right-angular solid.



Rule. Multiply the length, breadth, and depth together.

Let the length=20, width=12, height=10.

$$20 \times 12 \times 10 = 2400.$$

ART. 75. To find the solidity of a cylinder or prism.

Rule. Multiply the area of the base (or end), by the length.

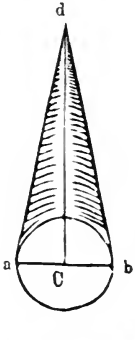
Note. Find the area of the base by previous rules, according to its shape.

ART. 76. To find the solidity of a solid wedge.

Rule. Multiply the area of the base by half the perpendicular length.

ART. 77. To find the solidity of a pyramid or cone.

Rule. Multiply the area of the base by one-third of the altitude.



Let a b=14, c d=24.

$$\begin{array}{l|l} \cancel{7} & \cancel{14}\ \ 2 \\ & 22 \\ \cancel{4} & 14 \\ \cancel{3} & \cancel{24}\ \ 2 \\ \hline & 1232 \ \textit{Ans.} \end{array}$$

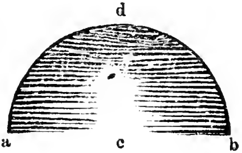
ART. 78. To find the solidity of the frustrum of a pyramid, or cone.

Rule. Add the areas of the upper, the lower, and the middle bases together, (the middle base is found by multiplying the upper and lower bases together, and extracting the square root of the product,) and multiply the sum by one-third of the altitude.

Art. 79. To find the solidity of a globe.

Rule. Multiply the area by one-sixth of the diameter; or, multiply the cube of the diameter by $\frac{11}{21}$.

Art. 80. To find the solidity of a spherical segment.



Rule. Add the square of the height to three times the square of the semi-diameter of the base, and multiply the sum by the height, and by $\frac{11}{21}$.

Let a b$=$10, c d$=$4.

$5 \times 5 \times 3 + 4 \times 4 = 91$. $91 \times 4 \times \frac{11}{21} = 190\frac{14}{21}$ *Ans.*

SQUARE ROOT.

Art. 81.—*Rule.* 1. *Separate the given number into periods of two figures each, commencing with units.*

2. *Find the greatest root in the left hand period, and place it on the right. Subtract its square from the first period, and to the remainder bring down the next period; and make a dividend of the remainder, with the first figure of the period annexed.*

3. *Double the root for a divisor, and set the quotient in the root, and to the right of the divisor.*

4. *Multiply and subtract as in division, and proceed, as before, until all the periods are brought down, when periods of noughts may be annexed to obtain decimals.*

1. What is the square root of 55225?

```
      5'52'25 (235=root
      4
  43) 1 52
      1 29
 465)   23 25
        23 25
```

2. What is the square root of 15625? *A.* 125.

3. What is the square root of 5'35.92'25?

Ans. 23.15.

CUBE ROOT.

ART. 82.—*Rule.* 1. *Separate the given number into periods of three figures each, commencing with units.*

2. *Find the greatest root in the left hand period, place it on the right, and subtract its cube from the left hand period.*

3. *To the remainder bring down the next period, and make a dividend of the remainder and the first figure of the period annexed.*

4. *Multiply the square of the root by 3, for a defective divisor. Place the quotient in the root, and its square to the right of said divisor, supplying the place of tens with a cypher, if the square be less than ten.*

5. *Complete the divisor by adding to it the product of the last figure in the root by the rest, and by 30.*

6. *Multiply and subtract as in division, and bring down the next period.*

7. *Find the next defective divisor, by adding to the last complete divisor the number which completed it, with twice the square of the last figure in the root, and proceed until all the periods are brought down, when decimals may be found by annexing periods of noughts.*

1. What is the cube root of 9663597 ?

	9'663'597(213
	8
2×2× 3=1201	1663
1×2×30= 60	
Complete div....... 1261	1261=*subtrahend.*
1×1× 2= 2	
Defect. div'r....132309	402597
21×3×30=1890	
Complete div ...134199	402597=*subtrahend.*

2. What is the cube root of 1953125?

Ans. 125.

3. What is the cube root of 33131834.347032?

Ans. 321.18.

We shall facilitate the rules of square and cube root by the following properties:

1. The product of two square numbers is a square number.
2. The quotient of two square numbers is a square.
3. The product of two cube numbers is a cube.
4. The quotient of two cube numbers is a cube.

Table of Squares and Cubes.

Nos.	1	2	3	4	5	6	7	8	9	10
Squ'rs.	1	4	9	16	25	36	49	64	81	100
Cubes.	1	8	27	64	125	216	343	512	729	1000

(See Art. 30.) In order to work with speed and alacrity, attention and tact are necessary on the part of the learner.

1. What is the side of a square piece of land containing 360 acres?

Instead of multiplying by 160, and extracting the square root, according to the common method, we remove a nought from one to the other, making them both squares, whose roots are 60 and 4, or 6 and 40.

Multiply the roots together. $60 \times 4 = 240$ *Ans.*

2. What is the square root of the product of 32 and 128?

$$\begin{array}{r|l} \cancel{16} & \cancel{32}\quad 2 \\ & \underline{\cancel{128}\quad 8} \\ & \sqrt{16}=4\times16=64 \ \textit{Ans.} \end{array}$$

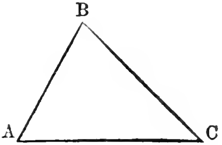
Here it will be observed that both numbers are divided by the factor 16, and the root of the product of the quotients multiplied by the factor.

Note. We should examine the table closely, so as to recognize a square or cube as soon as seen. The principle of removing noughts, or using factors, as explained in square root, is also applicable to cube root.

APPLICATION OF SQUARE AND CUBE ROOT, WITH MISCELLANEOUS MATTER.

To find the area of a scalene triangle.

Rule. From the half sum of the three sides, take the three sides severally, and extract the square root of the product of the three remainders and half sum.



1. Let A B$=5$
   Let B C$=6$
   Let A C$=7$

$$5+6+7\div 2=9.$$

$9-5=4,\ 9-6=3,\ 9-7=2.$

$$4\times 3\times 2\times 9=\sqrt{216}=14.696+ \quad \textit{Ans.}$$

2. What is the area of a scalene triangle, whose sides are 13, 14 and 15, respectively?
Ans. 84.

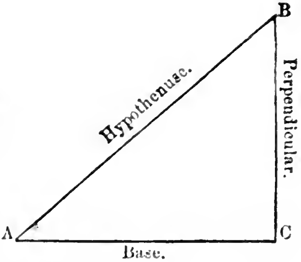
3. Given, the area of a circle 1296, to find the side of a square equal in area.

$$\sqrt{1296}=36. \qquad \textit{Ans. } 36.$$

Given, the area of a circle, to find the diameter.

Rule. Divide the area by $\frac{11}{14}$, *and extract the square root of the quotient.*

In any right angled triangle, (see adjoining figure, right-angled at C), when one leg and the hypothenuse are given, to find the other leg.



Rule. Multiply the sum by the difference, and extract the square root of the product.

When the two legs are given, to find the hypothenuse.

Rule. To double their product, add the square of their difference, and extract the square root of the sum.

4. What is the base of a right-angled triangle, whose hypothenuse is 10, and perpendicular 6?

$$\sqrt{10+6\times 10-6}=8 \textit{ Ans.}$$

5. Given, the perpendicular 3, and base 4, to find the hypothenuse.

$$\sqrt{3\times 4\times 2+1^2}=5 \textit{ Ans.}$$

Scholium 1. The product of the sum and difference of two numbers, is equal to the difference of their squares.

Scholium 2. Double the product of two numbers, plus the square of their difference, is equal to the sum of their squares.

6. A. and B. start from the same point; A travels due east 24 miles, and B. due north 18 miles; how far are they apart? *Ans.* 30 M.

7. Two men start from the same point; one travels 15 miles due south, the other 25 miles south-east; how far are they apart? *Ans.* 20 M.

8. What is the mean proportional between 4 and 9?

$$\sqrt{9\times4}=6 \text{ } \textit{Ans.}$$

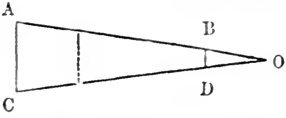
9. What is the area of a parallelogram, whose diagonal is 50, and the sides are as 3 to 4?

$$3^2+4^2=25 : 50^2 :: 3\times4 : [\,]=1200 \text{ } \textit{Ans.}$$

10. What are the sides of a parallelogram, whose area is 1200, and proportional of sides as 3 to 4? *Ans.* 30 and 40.

$$3\times4 : 1200 :: 3^2 : [\,].=900. \quad \sqrt{900}=30.$$
$$3\times4 : 1200 :: 4^2 : [\,].=1600. \quad \sqrt{1600}=40.$$

11. Given, the dimensions of a plank, 20 feet long, 18 inches wide at one end, and 6 at the other, to find how long the smaller end must be to contain half the number of square inches in the plank? *Ans.* 12.36.



Solution. Draw A B D C, and produce A B and C D to meet in O, making the triangle A O C 30 feet in length, containing $22\frac{1}{2}$ square feet; the triangle B O D, 10

feet in length, containing $2\frac{1}{2}$ square feet. Then in the triangle A O C, we only have to obtain a distance from O, sufficient to contain half the plank A B D C, plus $2\frac{1}{2}$ feet.

$22\frac{1}{2}-2\frac{1}{2}=20$, No. of sq. ft. in the plank.
$20\div2=10$, half the No. of sq. ft. in the plank.
$10+2\frac{1}{2}=12\frac{1}{2}$.
$22\frac{1}{2} : 12\frac{1}{2} :: 30^2 : [\,]=500$.
$\sqrt{500}=22.36-10=12.36$, length of small end.

Note. Areas are to each other as the squares of their similar sides.

12. By Rev. J. P. Murrel.

How long a line, do you suppose,
Would just the amount of land inclose,
That one can see on level ground,
Just from the cent'r, by turning round;
The eye about six feet in height,
And nothing to obstruct the sight?

Solution: *Ans.* $18\frac{6}{7}$ M.

$6\div2=\sqrt{3+6}=3R\times2=6D\times\frac{22}{7}=18\frac{6}{7}$ circumf.

13. If an eye be elevated 24 feet, how far, on level ground, can an object be seen?

$24\div2=12$. $\sqrt{24+12}=6$ M. *Ans.*

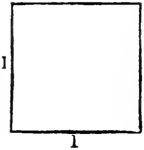
14. If an object is seen 9 miles, how high is the eye elevated? *Ans.* 54 ft.

$9^2\times\frac{2}{3}=54$ ft.

15 By Rev. J. P. Murrel.

Which will inclose the most ground,
A fence made square, or one made round,
Two pannels to each rod of land,
Ten rails in each, we understand;

And ev'ry rail in each suppose
To just an acre of land inclose.
The next thing is to tell exact,
How many acres in each tract?



Ans. $\begin{cases} 1024000, \text{ No. acres in sq're.} \\ 804571\frac{3}{7}, \text{ " " circle.} \end{cases}$

$\frac{1}{160}$=area of 1 rod square, which takes 80 rails to fence it.

Statements. $\begin{cases} \frac{1}{160} : 80 :: 80 : [\,] = 1024000. \\ 14 : 11 :: 1024000 : [\,] = 804571\frac{3}{7}. \end{cases}$

Note. One side of the square equals the diameter of the circle.

16. In what time will any sum of money triple itself, at 5 per cent? *Ans.* 40 yrs.

$$3-1 \times 100 \div 5 = 40.$$

17. In what time will any sum of money double itself, at 6 per cent? *Ans.* $16\frac{2}{3}$ yrs.

$$2-1 \times 100 \div 6 = 16\tfrac{2}{3}.$$

18. If I pay \$50 apiece for 7 mules, and the same amount for horses at \$70 apiece, and sell them at an average of \$60 apiece; do I gain or lose? *Ans.* \$20 gain.

19. If I purchase a number of pears at 3 cts. apiece, and pay the same amount for oranges at 6 cents apiece, and sell them all at an average of 4 cents, what do I gain or lose? *Ans.* \$0.00.

20. A horse in the midst of a meadow, suppose,
Made fast to a stake by a cord from his nose,
How long must this cord be, that feeding all 'round,
Permits him to graze just two acres of ground?

Ans. 10.0904+.

$160 \times 2 \div \frac{1}{14} = 407.272727$.

$\sqrt{407.272727} = 20.1809$ = diameter of circle.

Take half for radius, or cord.

21. A snail, climbing a pole 20 feet high, ascends 8 feet per day, and falls back 4 feet at night; how many days will it take him to reach the top? *Ans.* 4 days.

Statement—1 : 4 :: [] : 20—4=4.

Note. When an example and the solution are given, the solution is applicable to all similar examples.

22. A. can do a piece of work in 4 days, B. in 6 days; in what time can they both do it working together? *Ans.* $2\frac{2}{5}$ days.

$4 \times 6 \div 4 + 6 = 2\frac{2}{5}$.

23. As I was beating on the forest ground,
Up starts a hare before my two grey hounds,
The dogs being light of foot, did fairly run
Unto her fifteen rods, just twenty-one;
The distance that she started up before
Was four-score, sixteen rods, just, and no more.
Now, I would have you unto me declare,
How far they ran before they caught the hare?

D	96	H
21		15

Ans. 336 rods.

21—15 : 96 :: 21 : [].=336.

24. The hour and minute hands are exactly together at 12 o'clock; at what time will they next be together?

$11 : 12 :: 1 : [\,]$.=1h. 5min. $27\frac{3}{11}$sec.

25. The hands of a clock are together between 5 and 6; what is the time?

$11 : 12 :: 5 : [\,]$=5h. 27min. $16\frac{4}{11}$sec.

26. The hour and minute hands are directly opposite; the minute hand between 4 and 5, the hour between 10 and 11; what is the time?

Ans. 10h. 21m. $49\frac{1}{11}$sec.

Note. It is the same time past 10 as it would be past 4, were the hour and minute hands together between 4 and 5.

27. The time past noon is equal to $\frac{1}{3}$ of the time past midnight; what is the time?

Ans. 6 o'clock.

Denominator—numerator : numerator :: 12 : [].

2. The time past noon is equal to $\frac{1}{3}$ of the time till midnight; what is the time?

Ans. 3 o'clock.

Denominator+numerator : numerator :: 12 : [].

28. When first the marriage-knot was tied,
Between my wife and me,
My age did hers as far exceed
As three times three does three.
But after ten, and half ten years,
We man and wife had been,
Her age came up as near to mine,
As eight is to sixteen.

Now, tyro, skilled in numbers, say,
What were our ages on wedding-day?

Ans. $\begin{cases} 45 = \text{his age.} \\ 15 = \text{hers.} \end{cases}$

Solution. $\begin{cases} 3 \div 3 = 1. \quad 1 \times 15 = 15. \\ 9 \div 3 = 3. \quad 3 \times 15 = 45. \end{cases}$

29. A father gave to his son $\frac{3}{7}$ of his whole estate; to his daughter $\frac{5}{8}$ of the remainder, and the remaining part to his widow. The son received $75 more than the daughter. Required, the share of each.

Ans. $\begin{cases} \$450 = \text{son's share.} \\ \$375 = \text{daughter's.} \\ \$225 = \text{widow's.} \end{cases}$

30. If a beam, 10 ft. long, 5 in. wide, and 2 in. deep, will bear up 100 lbs., how many lbs. will another support, that is 15 ft. long, 6 in. deep, and 3 in. wide, the support being at the end. *Ans.* 360 lbs.

$$\left.\begin{matrix}2\\2\\5\end{matrix}\right\} \div 10 : 100 :: \left.\begin{matrix}6\\6\\3\end{matrix}\right\} \div 15 : [\,].$$

$$\begin{array}{r|l} \cancel{2} & \cancel{10} \\ \cancel{2} & \cancel{6}\ \cancel{3} \\ \cancel{5} & 6 \\ \cancel{3}\ \cancel{15} & 3 \\ & \cancel{100}\ 20 \\ \hline & 360 \end{array}$$

To find the least common multiple of fractions.

Rule. Find the least common multiple of the numerators, and divide it by the greatest common divisor of the denominators.*

* The process is the same as in Art. 13

31. Find the least common multiple of $\frac{2}{3}$, $\frac{4}{5}$, and $\frac{6}{7}$. *Ans.* 12.

The least common multiple is 12.
The greatest common divisor is 1.

Thus $12 \div 1 = 12$.

32. By Rev. J. P. Murrel.

Suppose two walls erect should stand,
Across a street on either hand;
Now, if a pole should stand upright,
Close to the one of the same height,
If foot be drawn twelve feet in street,
And top slips down about two feet;
Then if the top be turned to fall,
So as to strike the other wall
Below its summit just six feet—
What are their heights, and width of street?

Ans. { 37 ft. height of walls. 32.2 ft. width of street.

Solution.

$12^2 + 2^2 \div$ double the distance the pole slipped down $= 37$ ft., height.

$$37 - 6 + 37 \times 37 - 31 = 408.$$

$$\sqrt{408} = 20.2 (\text{nearly}) + 12 = 32.2, \text{ width.}$$

33. A man has 60 lbs. of wool, worth 25 cts. per lb., which he wishes carded. The carder has 5 cents per lb. for carding, and takes the toll in wool before carding it; how many lbs. must he take? *Ans.* 10 lbs.

$$60 \times 5 \div 25 + 5 = 10.$$

Falling Bodies.

Rule. Multiply the square of the time by $16\frac{1}{12}$ feet, (the distance a body will fall in the first second).

34. To what height must a stone be raised to require it 4 seconds to reach the ground?

Ans. $257\frac{1}{3}$ ft.

$$4^2 \times 16\tfrac{1}{12} = 257\tfrac{1}{3}.$$

35. What is the depth of a well, to the bottom of which a stone would be 3 seconds in falling? *Ans.* $144\frac{3}{4}$ ft.

36. How high above the earth's surface must a body be raised to lose $\frac{1}{3}$ of its weight?

Ans. 898.97.

Rule. Denominator—numerator : denominator :: square of the earth's semi-diameter ∴ to the square of the distance from the center *of the earth.*

$3-1 : 3 :: 4000^2 : \sqrt{24000000} = 4898.97 - 4000 = 898.97$ miles.

37. By Rev. J. P. Murrel.

If a man o'er head, in a balloon,
Should fire a level gun at noon,
How high is he, if ball and sound,
At the same time, should strike the ground?

Ans. $15\frac{1}{3}+$miles.

Note. Sound flies 1142 feet per second.

$$1142^2 \div 16\tfrac{1}{12} = \text{No. of feet.}$$

38. Divide 100 apples between John and James, so that John will have $\frac{1}{3}$ more than James.

$100 \div 1 + 1\frac{1}{3} = 42\frac{6}{7}$. *Ans.* $\begin{cases} 42\frac{6}{7} = \text{James.} \\ 57\frac{1}{7} = \text{John.} \end{cases}$

39. If the fourth of 20 be 3,
What will the fifth of 30 be?

Ans. $3\frac{3}{5}$.

40. If the third of 7 be 3,
What will the sixth of 20 be?

Ans. $4\frac{2}{7}$.

41. Two pence is what part of $\frac{2}{3}$ of 3 pence? *Ans.* the whole.

42. What is nothing, twice yourself, and 50? *Ans.* O. W. L.

43. Four-fifths of $15 are six-tenths of how many thirds of $21? *Ans.* $2\frac{6}{7}$.

44. Four-sevenths of 42 are $\frac{2}{3}$ of how many times that number of which $\frac{1}{3}$ of 9 is $\frac{3}{4}$? *Ans.* 9.

45. If A., B., C. and D. start from the same point around a circular island, 80 miles in circumference, in how many days will they next be together, if A. travels 4 miles per day, B. 8, C. 20, and D. 28? *Ans.* 20.

Rule. Divide the distance round the island by the greatest common divisor of distances traveled.

46. By Rev. J. P. Murrel.

Suppose a horizontal plane,
On which did stand a stalk of cane,
The height of which I took quite neat,
And found to be one hundred feet.
Soon as I did this measure take,
A blast of wind this cane did break.

The top of it did strike the ground
Some ten yards from the base, I found;
One end of which did still remain,
Just where the wind did break the cane.
Now, can you all these measures take,
And tell how high the cane did break?

Ans. $45\frac{1}{2}$ ft.

$$100^2-30^2\div100\times2=45\frac{1}{2}.$$

47. Two men, A. and B., leave Iowa City. A. travels due east, 39 miles, to Berlin; B. due south to St. Francisville, thence to Berlin, and says he has traveled 120 miles since he left Iowa City. They both start a due south course, and after having passed the latitude of St. Francisville, travel 83 miles to Jefferson Barracks. They then turn a due west course to a point exactly south of St. Francisville. They here part. B. passes directly to Iowa City. A. continues his course due west to Jefferson City, thence to St. Francisville, and says, since he parted with B. the last time, he has traveled 100 miles. Returning to Jefferson City, he inquires how far he and B. are separated. *Ans.* 137.544+M.

48. What is the length of a trout, whose head is 3 inches long, his tail as long as his head and $\frac{2}{5}$ of his body, and his body as long as his head and tail together? *Ans.* 20 in.

Head.	Body.			Tail.	
3	3	3	4	4	3

49. Edwin bought 5 pears for 5 cents; Charles bought 3 for 3 cents; being afterwards joined by James, the three made a meal of the 8 pears. On leaving, James pays them 8 cents, of which

Charles claims 3 cents, as he furnished 3 pears. How, in equity, should the 8 cents be divided?

Ans. { Charles, 1 ct.
Edwin, 7 cts.

50. By Rev. J. P. Murrel.

In partnership, we understand,
Two brothers bought a piece of land,
Two hundred acres when survey'd,
And each four hundred dollars paid.
One end being richer than the other,
The elder said to the younger brother,
I'll take the end that 's not so poor,
And pay a half a dollar more
Per acre, if you will agree,
To let that end belong to me.
To which the younger thus replied,
I will, if you 'll the land divide.
I will, he said, and at it went;
But, after he some days had spent,
And found it did his soul perplex,
He called on his surveyor next,
Who labored hard, but could not quite
Make land and money come out right;
Then threw it down with grief and pain,
Declaring he'd ne'er try again.
Since that, this sum has traveled round,
To see if any could be found
Who could this piece of land divide,
As elder brother did decide;
Likewise how much each man must pay
Per acre, in his own survey.
Now, reader, as it's come to you,
Take hold, and see what you can do!

Ans. { Elder brother's, $93\frac{3}{4}+$acres.
Younger brother's, $106\frac{1}{4}$ acres, nearly.
Price of elder brother's, \$4.26.5+.
Price of younger brother's, \$3.76.5+.

For the solution of the above:

Rule. Find the cost of the whole number of acres, at the difference between the prices per acre,

which subtract from the amount paid for the whole land; to the square of the remainder, add the product obtained by multiplying the cost of the whole number of acres (at the difference between the prices per acre), by four times the whole sum paid by him who paid least per acre; extract the square root of the sum; to the result add the remainder that was squared, and divide the sum by twice the whole number of acres, for the price per acre paid by him who paid least per acre. Having this, other requirements of the question are easily found.

51. By Rev. J. P. Murrel.

If round a point two wheels you start,
By axle kept five feet apart,
The height of inner wheel complete,
Supposed to be about three feet—
To form a circle would require
The outer wheel a little higher;
How much higher would you suppose,
That inner might an acre inclose?

Ans. .1274+ft.

117.728 ft.=radius of one acre.

117.728 : 117.728+5 :: 3 : []. 3,1274+ft. height of larger wheel.

52. By Rev. J. P. Murrel.

Suppose a cart drawn once around
A level circular piece of ground,
Diameter of wheels to be,
In inches, each just sixty-three.
Now, if the axle of this cart
Should keep the wheels five feet apart,
How many times will one turn round
More than the other, once round this ground,
If inner track should just inclose,
Five thousand acres we'll suppose?

Ans. $1\frac{9}{21}$.

$5\times 2\times\frac{22}{7}$ = difference of peripheries.
$63\div12\times\frac{22}{7}$ = circumference of wheels.

$$\frac{220}{7}\div\frac{33}{2}=1\frac{19}{21}.$$

Note. The No. of acres has nothing to do with the solution.

53. How wide must a walk be around a rectangular garden, 24 yds. long, and 16 yds. wide, to contain as much land as the garden?

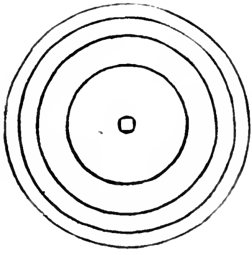
Ans. 4 yds.

Solution.

$24\times16\div4=96.$
$24+16\div4=10.\quad 10^2+96=\sqrt{196}=14.$
$14-10=4$, width of walk.

54. By Rev. J. P. Murrel.

Four men, A., B., C. and D.,
In partnership did buy
A grindstone, which they did agree
To grind away, all but the eye.
How deep in radius must each grind,
To have an equal share;
Diameter five feet they find,
And eye three inches square?



A. 4.00+in. 1st man.
4.73+in. 2d man.
6.15+in. 3d man.
12.99+in. 4th man.

$60^2 \times \frac{11}{14} = 2828\frac{4}{7}$ = area of stone.
$3^2 + 3^2 \times \frac{11}{14} = 14\frac{1}{7}$ = area of circle about the eye.
$4\overline{)2814\frac{3}{7}}$ = area to be ground away.
$703\frac{17}{28}$ = area of one man's share.
$\sqrt{2828\frac{4}{7} - 703\frac{17}{28} \div \frac{11}{14}} = 52.+$. $60 - 52 \div 2 = 4$ = radius of first man's share.

Note. For the other depths of radii proceed in the same manner.

55. Bought 2 watches for $20 each, and sold one at 25 per cent. gain, the other at 20 per cent. loss; did I gain or lose? *Ans.* $1 gain.

56. When 5 pears are worth 7 peaches, and 12 peaches 15 apples, and 21 apples 24 damsons, how many pears can I have for 36 damsons? *Ans.* 18 pears.

$$\begin{array}{r|l} \cancel{7} & \cancel{5} \\ \cancel{15} & \cancel{12} \\ & \cancel{21} \\ \cancel{2}\ \cancel{24} & \cancel{36}\ \ 18 \\ \hline & \quad 18 \text{ pears.} \end{array}$$

57. By Rev. J. P. Murrel.

How large a field would be required,
Inclosed by fence both staked and ridered;
Two pannels to each rod of land,
Ten rails in each we understand,
So that the fence may just inclose
As many acres, we'll suppose,
As rails, and stakes, and riders there,
The field itself to be a square?

Ans. 1730560 acres.

58. By Rev. N. C. DeWitt.

A wealthy man a daughter had,
A son likewise — a sprightly lad —
To whom he gave a piece of land,
Which was a square, we understand;
On ev'ry side the distance found
Was just four hundred rods of ground.
The next thing is the daughter's share,
Which must be *round*, and not a *square*;
How long a line, do you suppose,
Would just the daughter's land inclose,
And she the same amount obtain
That's in her brother's large domain?

Ans. 1418 rods.

Solution: $400 \times 3.545 = 1418.000$.

59. By Rev. J. P. Murrel.

A youth who lived a single life,
Set out at last to hunt a wife,
So to a house he did repair,
To see if he could find one there.
Directly after he stepped in,
With Miss —— a courtship did begin,
When she embraced the chance to find
What were the powers of his mind.
"My brother and myself," she said,
"Were all the heirs my father had;
And now, kind sir, please understand,
We both were in a foreign land."
Then with a plaintive voice she sighed,
"Pa heard that one of us had died."
Before she closed she added still,
"While we were there Pa made his *will*,
Which did for Ma and me provide,
If I had lived and brother died:
Two-thirds of his estate should be
Secured to Ma, one-third to me.
Had brother lived and I had died,
For them the will did thus provide:
Two-thirds were given to my brother,
And only one-third left to mother.
Now, as we both are living still,
And must be governed by the will,

Which does my mother's share express,
About three hundred dollars less
Than it would be if I had died,
And they should by the will abide.
And now, kind sir, please calculate,
What is the sum of Pa's estate;
Likewise how much each share will be,
According to the will's decree?
Unless you do all these decide,
I'll not consent to be your bride, (*sure.*)

$$\textit{Ans.}\begin{cases}\text{Mother's,} & \$1800.\\ \text{Daughter's,} & \$\ 900.\\ \text{Brother's,} & \$3600.\\ \text{Whole estate,} & \$6300.\end{cases}$$

Solution:

$1 = \text{D.}$
$2 = \text{M.}$
$\underline{4 = \text{B.}}$
$7 \div 3 = 2\frac{1}{3}$, had daughter died.
$\underline{2}$, all being alive.
$\frac{1}{3} : 300 :: 2 : [\,] = \$1800$ M's.

60. By Rev. S. H. Hodges.

A neighbor asked me for the time,
And as I love to speak in rhyme,
I told him it was after ten,
And both the hands together then.
Pray tell me now the time precise,
By any means you can devise.

Ans. 10h. 54min. $32\frac{8}{11}$ sec.

61. By Rev. S. H. Hodges.

If three-elevenths of the age
Of one who is a noble sage,
By twenty-eight should be increased,
And one year from the last released,

And one-eleventh of a year
To the remainder added here,
And of a year elevenths three
Be now subtracted, you will see,-
You'll have one half the sage's years;
Now, what is all his life of cares?

Ans. **118 years.**

62. By E. P. Thompson.

Suppose a mirror sixteen inches wide,
In inches long precisely twenty-four,
The frame of which the owner does decide,
In superfice must equal it—no more.
How wide a frame, pray unto me declare,
That each shall equal be, in inches square?

Ans. **4 inches.**

63. By Rev. J. P. Murrel.

A youth who lived a lonely life,
Concluded he must have a wife;
He sought a fair one for his bride,
Whose father just before had died.
The fair one's heart and hand were gained,
But something to be done remained—
The maiden's mother must consent,
Before the matter further went.
On being asked, the mother said,
"Why do you bother thus my head?—
To you, sir, it is clearly known
That I have just five daughters grown.
Their father's will says, 'my *first four*
Must have *twelve thousand*, and no more;
To my *last four* I do declare
Eleven thousand is their share;
To my *last three* and *first* I give
Ten thousand dollars, if they live;
As to my *last*, and my *first three*,
Nine thousand shall their portion be;
To all, except the *third* alone,
I give *eight thousand*—now, I'm done.'
Now," said the mother, "if you tell
The *third* one's part, I'll give you Nell."

Ans. **$4500.**

Solution:

12000=A.B.C.D.
11000=B.C.D.E.
10000=A.C.D.E.
9000=A.B.C.E.
8000=A.B.D.E.
4)50000=four times A.B.C.D.E.
12500=A.B.C.D.E.
8000=A.B.D.E.
4500=C. (or Nell.)

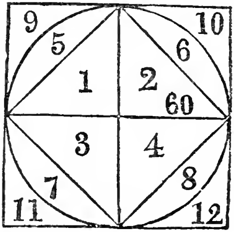
64. Suppose a cone to stand upright,
Which is one foot exact in height,
How high, 'bove base must a line be,
That will divide it equally?

Ans. 2.476 inches.

$\sqrt[3]{12^3 \div 2}=9.524.$ $12-9.524=2.476.$

65.

By Rev. S. H. Hodges.



A farmer has a square of fertile land,
And in the center all his buildings stand;
His land he has determined to divide
Among twelve sons, as he can best decide.
He first proceeds to draw a circle round,
With area as broad as all his ground;
And next he doth proceed to draw a square
Whose angles in the circle's bound'ry are;
He then four radii doth draw complete,
Which in four points with square and circle meet;
Each radius in length is sixty poles,
And this amount alone the sum controls.

The lines now made divide the farmer's ground,
So all the sons may have their portions round.
Now, tell me how much land there is in all,
And how much will within the circle fall—
How much within the inner square will be;
And also each son's portion tell to me.
But bear in mind, the first four each must take
One-fourth the inner square, his part to make.
Unto the second four the segments give,
That they within the circle's rim may live.
As to the last four, give them for their lots
The outside sections — smallest, richest spots.

In the whole track,	90	acres
In the circle,	$70\frac{5}{7}$	"
In the inner square,	45	"
First four sons, next the farmer's, each,	$11\frac{1}{4}$	"
Second four, in the segments, have each	$6\frac{3}{7}$	"
Last four, in the outside sections, each,	$4\frac{23}{28}$	"

66. Three men, A, B, and C, being employed to perform a certain piece of work for $105, A and B are supposed to do $\frac{3}{11}$ of it, A and C $\frac{5}{13}$, and B and C $\frac{2}{7}$. They are paid proportionally; please divide their pay for them as it should be.

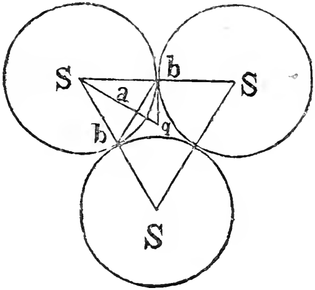
67. A stick of timber 30 feet long, of uniform thickness and breadth, is to be lifted by 3 men, one at one end, and the other two holding a hand-spike near the other. How far from the end must they be placed that each of the three may raise an equal portion of the stick?

68. A man 6 feet high traveled round the earth. How much further did his head go than his feet?

69. By Rev. J. P. Murrel.

There is a Quaker, we understand,
Who for three sons laid off his land,
And made three equal circles meet,
So as to bound an acre neat.
Just in the center of that acre,
Is found the dwelling of the Quaker;
In centers of the circles round,
A dwelling for each son is found:
Now, can you tell, by skill and art.
How many rods they live apart?

Ans. { Distance from son to son, 63 rods.
" " father to son, 36.372 rods.



Solution:

$\sqrt{160\times 6.20156}=31.5$(nearly)=S b.
$31.5\times 2=63=$S S.
$31.5\div 2=15\frac{3}{4}=$a b.
$\sqrt{\frac{3}{4} : \frac{1}{2} :: 15\frac{3}{4} : [\,]}=9.093=$a q.
$\sqrt{31.5^2-15.75^2}=27.279=$S a.
27.279-1 9.093=36.372=q S.

70. By E. P. Thompson.

The author of this work I chanced to ask,
"Tell me, sir, if you please, how old you are?"
And he replied, "Let me impose this task,
Which, when performed, it will my years declare:
If to my age one-fourth score added be,
And of that sum the square root you extract,
And add it to the sum, you then will see
That you will have just one score, ten, exact."
You, who with numbers do your thoughts engage,
Pray tell me, if you can, what is his age. *Ans.* —.

71. What number is that, to which, if you add the square root, the sum will be 42? *Ans.* 36.

Rule. To the sum (42), add one-fourth, and extract the square root, from which take one-half for the square root of the number.

72. By E. P. Thompson.

"Pray," said a lover to his "gal,"
"Will you not be my bride?
My heart, my hand, my all, are thine,
Whatever may betide."
To which, that she might test his skill,
The pretty thing replied:
"A gent was wont to visit where
Three sisters did reside;
And so, ere starting there one day,
Within his pockets wide
He put of pears a number there,
And did them thus divide:
To Kate he gave one-half he had,
And half a one beside;
The half then left, and half a pear,
For Mol he did provide;
Half then remaining, plus one-half,
For Puss he set aside;
He'd then one left. Now, tell me, sir,"
The gentle maiden sighed,
"How many pears in all had he,
To 'mong the three divide?
My answer must, shall be delayed,
Till this you do decide. *Ans.* —

RECOMMENDATIONS.

I HAVE examined, with much interest, many portions of Mr. I. N. WILCOXSON's "Arithmetical Vade Mecum," (in manuscript), and find myself *decidedly* favorable to the work. It will doubtless prove, not only a *convenient*, but also a very *useful companion*, to all calculators into whose hands it may fall. I have been a teacher of youth, the most of my time, for about fourteen years; during which time, I have examined and used a considerable number of Arithmetics; but, for *conciseness*, *perspicuity*, and *practical utility*, I feel constrained to praise the "Vade Mecum" more than all of them. The young authoı of this new work deserves the thanks and patronage of many, for the great improvement he has made in the philosophy and practice of *Arithmetic*. I am strikingly impressed with the idea of our author—by his *untiring energy* and *intense study*—becoming an instructor of *instructors!* Let every instructor, merchant, mechanic, student, and farmer, procure a copy of this new work, *study* it, and *apply its rules in practice*, before he suffers himself to speak or think against it.

S. H. HODGES.

Barren County, Kentucky.

THE following is from the pen of JAMES G. HARDY, Lieutenant Governor of Kentucky:

GLASGOW, KY., *Sept.* 29, 1855.

I have briefly examined the "*Arithmetical Vade Mecum*," prepared by ISAAC N. WILCOXSON, Esq. The principles of "Cause and Effect," or a separating the order of producing from the parts produced, are worthy the consideration and patronage of the public, and I have no hesitation in saying that the work will be useful to business men.

JAS. G. HARDY.

A SUPPLEMENT

TO THE

ARITHMETICAL VADE MECUM:

GIVING AN EXPLANATION AND SHOWING THE APPLICATION OF A NEW DIAGRAM; BY MEANS OF WHICH WE READILY APPLY CAUSE AND EFFECT TO NUMERICAL CALCULATION.

THERE is but one rule by which every arithmetical question of a practical nature in arithmetic may be scientifically and philosophically analyzed, stated, and solved; it being founded upon an axiom in natural philosophy, "That equal causes produce equal effects, and that effects are always proportionate to their causes." There are but two primal principles—increase and decrease—in numerical calculation; and, by the same statement, we increase or decrease according to the nature of the question. This may be learned by those of common capacity, who are attentive, and understand the first principles of calculation, in a very few days. There is contained in the following pages of the Supplement the labor and experience of years, teaching and investigating, exclusively, the new system.

I owe many thanks to Ed. Porter Thompson, now Principal of Rich Grove Seminary, a new and flourishing Institute on the turnpike seven miles north of Glasgow, Kentucky, in a delightful neighborhood, for much assistance rendered.

ISAAC N. WILCOXSON.

AUGUST 24, 1858.

SUPPLEMENT.

FIRST COUPLET. 1st Proposition. — SECOND COUPLET. 2nd Proposition.

—[MEANS.]—

Cause : Effect : : Cause : Effect.

(1st term.) (2nd term.) (3rd term.) (4th term.)

—[EXTREMES.]—

Question. WHAT is a diagram?—*Answer.* An arithmetical scheme.

Q. What system is here used?—*A.* Diagramic system of applying *cause* and *effect* to numerical calculation.

Q. What is the *rule*?

A. RULE:

Cause : (is to) *effect* : : (as) *cause* : (is to) *effect*.

NOTE.—The preceding is the RULE complete, but it may be read with blank terms, as follows:

Cause : (is to) *effect* : : (as) *cause* : [—]* (is to required) *effect*.

---

* Is the blank, and shows the place of the required term, or factor.

or,

Cause : (is to) *effect* : : [—] (as required) *cause* : (is to) *effect.*

or,

Cause : [—] (is to required) *effect* : : (as) *cause* : (is to) *effect.*

or,

[—] (Required) *cause* : (is to) *effect* : : (as) *cause* : (is to) *effect*. (See Art. 29.)

NOTE.—Sometimes there is only a factor of a term blank, or wanting; but this does not alter the rule, or statement, in the least; it would be as the following example in all the terms.

Cause : (is to) *effect* : : (as) *cause* : (is to) { factor *effect.* / [—] required fac. }

Q. What is a term ?—*A.* One member of a proportion.

Q. How many terms in the diagram ?—*A.* Four.

NOTE.—There may be a multitude of factors; there may be several in a term, as for example: If 3 men in 8 days build a wall 20 feet long, 12 feet high, and 2 feet thick; how many men would be required in 4 days to build another 24 feet long, 10 feet high, and 5 feet thick?

STATEMENT.

C.		E.		C.		E.
Men 3	:	length 20	: :	(—)	:	24
Days 8		height 12		4		10
		thickness 2				5

Here 3 and 8 are factors of the first term; 3x8=24 the term; 20, 12, and 2 are factors of the second term; 20x12x2=480 the term.

Q. What are they divided into ?—*A*. Means and extremes.

(See Art. 26 A. V. M. for answers to following questions.)

Q. What is ratio ?

Q. What is a couplet ?

Q. What terms compared constitute the first couplet ?

Q. What terms compared constitute the second couplet ?

Q. What are the means ?

Q. What are the extremes ?

Q. What is a proportion ?

Q. The product of the means must equal what ? (See Art. 27.)

NOTE.—If 4 yds. of cloth cost $12, 6 yds. will cost how much ?

STATEMENT.

C.		E.		C.		E.
Yds. 4	:	$12	::	6	:	(—)

After the statement is made, we consider the factors as abstract numbers. In the first couplet the effect is 3 times as large as its cause ; so must the second effect be 3 times as large as its cause—6x3=18 second effect. Then 18x4=12x6. The same reasoning applies to all the terms.

Q. What is a proposition ?—*A*. A statement in terms, one clause of a question containing two terms, as 4 yds. of cloth cost $12.

Q. How many propositions in every arithmetical question?—*A.* Two.

Q. What are they?—*A.* First and second.

Q. What kind are they?—*A.* Complete and incomplete.

NOTE.—The complete proposition is a model, an example, by which to arrange or complete the incomplete proposition. The complete governs the incomplete.

Q. Where is the first proposition arranged?—*A.* In the first couplet.

Q. How?—*A.* To suit convenience.

Q. If time is given, where is it placed?—*A.* In the same term with that that can produce action or agency.

Q. Where is the second proposition arranged?—*A.* In the second couplet.

Q. How?—*A.* Similar to the first.

Q. What is cause?—*A.* Action or agency.

Q. What is effect?—*A* That which is accomplished or follows action or agency.

Q. What are the guides in stating the question?—*A.* The elements or denominations in the question. (See Art. 31.)

Q. There must be the same number of elements or factors where?—*A.* In similar terms. (See Art. 31. N. B.)

Q. What must be in similar terms?—*A.* The same number of elements; also, the same number of factors, though one may be a blank factor.

Q. Causes must always be how?—*A.* Of the same or similar kind.

Q. And must be reduced to what before placed on the line?—*A.* To the same name or denomination.

Q. Where is the (—) blank placed?—*A.* In the place of the required term or factor; or in that term similar to that in which is placed the same or similar element or name in the arrangement of the complete proposition.

NOTE.—The observing student will notice that there is an action or agency that passes from the cause to the effect —a relationship shown—which will enable him to readily separate the factors of the cause from those of the effect. The philosophical way of stating the question is to place the acting term as cause; but, to find the true result or answer, it is immaterial which is placed as cause, so that the second is like the first.

Q. If the first or fourth term is blank, which are complete, means or extremes? and where are they placed?—*A.* The means are complete, and are placed upon the right as a dividend.

Q. Which are incomplete? and where are they placed?—*A.* The extremes are incomplete, and are placed upon the left as a divisor.

Q. If the blank is in either the 2nd or 3rd

term, which are complete, means or extremes? and where are they placed?—*A.* Extremes are complete and are placed upon the right as a dividend.

Q. Which are incomplete? and where are they placed?—*A.* The means are incomplete, and are placed upon the left as a divisor.

Q. Then what is involved to find the true result or answer to the question?—*A.* Nothing but simple multiplication and division.

Q. What is cancellation used for?—*A.* To shorten the work of multiplication and division.

INTEREST.

(See Art. 36 A. V. M.)

PERCENTAGE.

Cause : *effect* :: *cause* : *effect.*
100 : 100+*gain* ℘ *ct.* :: *cost price* : *selling price.*
or,
100—*loss* ℘ *ct.*

BARTER.

Cause : *effect* :: *cause* : *effect.*
1*st commodity* : 1 (*am't*) :: 2*d comm'y* : 1 (*am't*).
Its price : :: *its price.* :

DISCOUNT.

Cause :	*effect*	: :	*cause*	:	*effect.*
100 :	100+*interest of 100 for given time and rate* ℔ *ct.*	: :	*present worth*	:	*amount to be discounted.*

EXAMPLE.

What is the present worth and the discount of \$110 for 1 year and 8 months, at 6 per cent ?

Ans. Pres. \$100. Dis. \$10.

C.		E.		C.		E.
100	:	110	: :	(—)	:	110=100 pres.

1*y.* 8*m.*=20*m.*×6÷12=10+100=110.

MENSURATION TABLE.

(See Art. 46 A. V. M.)

SQUARE MEASURE.

Cause	:	*effect*	: :	*cause*	:	*effect.*
Factors of the unit of measure	:	*Unit of measure*	: :	*Length Width*	:	*No. of units.*

SOLID OR CUBIC MEASURE.

Cause	:	*effect*	: :	*cause*	:	*effect.*
Factors of the unit of measure	:	*Unit of measure*	: :	*Length Width Depth*	:	*No. of units.*

NOTE.—In square measure put length and width, and in solid measure put length, width, and depth, under the second cause, and reduce the first cause to the same denomination, then place it on the line for solution.

Q. What is the first proposition in interest?—*A*. 100, 1 year, and rate per cent.

Q. What is the second proposition?—*A*. Principal, time, and interest.

Q. What is the first proposition in percentage?—*A*. 100 and its amount.

Q. How do you find that amount?—*A*. By adding the gain per cent to, or subtracting the loss per cent from, 100.

Q. What is the second proposition in percentage?—*A*. Cost and selling price.

Q. What is the first proposition in barter?—*A*. First commodity, its price, and 1 (amount).

Q. What is the second proposition?—A. Second commodity, its price, and 1 (amount).

Q. What is the first proposition in discount?—*A*. 100 and its amount.

Q. How is that amount found?—*A*. By getting the interest on 100 for given time and rate per cent and adding it to the 100.

Q. What is the second proposition in discount?—*A*. Present worth and amount to be discounted.

Q. What is the first proposition in measurement?—*A*. The unit of measure and its factors.

Q. What is the second proposition?—*A*. The factors of the thing to be measured, and the number of units it contains.

NOTE.—By the foregoing questions the investigator can clearly see that there is only the one rule and principle necessary to analyze, state, and solve all questions coming up under the preceding specified rules. It may also be applied to any practical calculation in life.

MISCELLANEOUS QUESTIONS.

1. Add together $\frac{5}{6}$ $\frac{2}{3}$ $\frac{1}{4}$ $\frac{7}{9}$ $\frac{11}{12}$ and $\frac{1}{2}$.

3	6	5	30
2	3	2	24
2	4	1	9
3	9	7	28
	12	11	33
	2	1	18
Least com. denom.	36		$142 = 3\frac{17}{18}$

(See A. V. M.)

2. Add together $\frac{1}{2}$ $\frac{2}{3}$ $\frac{3}{4}$ $\frac{5}{6}$ $\frac{1}{12}$. *Ans.* $2\frac{5}{6}$.

3. From $\frac{7}{15}$ take $\frac{2}{5}$. *Ans.* $\frac{1}{15}$.

4. Multiply together $\frac{3}{20}$ $\frac{10}{5}$ $\frac{2}{3}$ $\frac{15}{3}$ $\frac{25}{4}$ $\frac{16}{20}$ and $\frac{6}{2}$.
Ans. 15.

5. Multiply together $\frac{3}{4}$ $\frac{7}{9}$ $\frac{2}{3}$ $\frac{4}{5}$ $1\frac{3}{7}$ 3 $1\frac{2}{3}$ and $1\frac{3}{7}$.
Ans. $2\frac{6}{7}$.

6. Divide $\frac{1}{2}$ by $\frac{1}{4}$. *Ans* 2.

7. Divide $\frac{1}{4}$ by $\frac{1}{2}$. *Ans.* $\frac{1}{2}$.

NOTE.—Multiplying by a fraction decreases the multiplicand; and dividing by a fraction increases the dividend.

8. Divide 4 by $\frac{1}{3}$. *Ans.* 12.

9. Divide $\frac{1}{2}$ by 5. *Ans.* $\frac{1}{10}$.

10. Divide $\frac{3}{8}$ of 4 by $\frac{6}{16}$ of 2, and multiply the quotient by $2\frac{6}{7}$ of $\frac{1}{4}$, and divide by $3\frac{1}{3}$ of $\frac{1}{7}$.
Ans. 3.

11. If 8 hats cost \$40, what will 6 hats cost?
Ans. \$30.

12. If 200 ℔ of pork cost \$10, how many ℔ can I have for \$40? Ans. 800 ℔.

13. If a pole 7 feet high, at noon cast a shadow 5 feet long, what is the height of a tree whose shadow measures 80 feet? Ans. 112 ft.

14. If 3 men in 10 days build 20 rods of fence, how long a time should be allowed 5 men to build 50 rods? Ans. 15 days.

15. If \$200 in 8 months gain \$10 interest, how many months will it take the same principal to amount to \$250? Ans. 40m.

16. Suppose 1 woman in $2\frac{1}{2}$ days sews the seams of two pair of pants, each of which average $4\frac{1}{2}$ seams 30 inches long and 12 stitches to the inch, how many days should it take 5 women to make 12 pair each of which averages $4\frac{3}{4}$ seams 48 inches long and 15 stitches to the inch? Ans. 5 days.

17. When 10 men in 6 days dig a trench 40 feet long, 5 feet wide, and 8 feet deep, that is of the hardness of 3, how many days ought it to take 80 men to dig another that is 400 feet long, 8 feet wide, 18 feet deep, and of the hardness of 4? Ans 36 days.

What is the interest of—

18. $235 for 30 days at 12 per ct.
Ans. 2.35

19. $350 for 60 days at 6 per ct.
Ans. 3.50

20. $700 for 90 days at 4 per ct.
Ans. 7.00

21. $360 for 2 days at 6 per ct.
Ans. 12cts.

22. $120 for 1y. 4m. 20d. at 6 per ct.
Ans. 10.00

23. What principal at interest for 8 months and 10 days, at 6 per ct., will draw $15 interest?
Ans. 360 00

24. At what rate per ct. will $900, in 1 year, 1 month and 10 days, draw $60 interest?
Ans. 6 per ct.

25. In what time will \$1000, at 6 per ct., draw \$100 interest? Ans. 20m.

How must the following articles be sold, that cost—

26. $8\frac{1}{3}$ cts. to clear 50 per ct.? Ans. $12\frac{1}{2}$ cts.

27. 10 cts. to clear 50 per ct.? Ans. 15 cts.

28. $12\frac{1}{2}$ cts. to clear 50 per ct.? Ans. $18\frac{3}{4}$ cts.

29. 80 cts. to clear 25 per ct.? Ans. \$1.00

30. 60 cts to clear $33\frac{1}{3}$ per ct.? Ans. 80 cts.

31. 10 cts. to clear 10 per ct.? Ans. 11 cts.

32. \$5 to clear 20 per ct.? Ans. \$6.

33. \$4 to lose 25 per ct.? Ans. \$3.

34. \$1 to lose 10 per ct.? Ans. 90 cts.

35. Pay 25 cts. for an article, and sell it for 30 cts., what per ct. is gained? Ans. 20 per ct.

36. Pay 5 cts., and sell at $8\frac{1}{3}$, what per ct. is gained? Ans. $66\frac{2}{3}$.

37. Pay 40 cts., and sell at 50, what per ct. is gained? Ans. 25.

38. Pay $12\frac{1}{2}$, and sell at 10, what per ct. is lost? Ans. 20.

39. Sold a horse for \$75, and lost 25 per ct., what did he cost? Ans. \$100.

40. Sold coffee at 22 cts. per lb. and gained 10 per ct., what did it cost? Ans. 20 cts.

41. Barter 8 ℔ of butter at $12\frac{1}{2}$ cts. per ℔ for domestic at 10 cts. per yard, how many yards must I have? Ans. 10 yds.

What is the present worth and discount of—

42. \$303 for 1m. 15d. at 8 per ct.?
Ans. P. W. \$300. D. \$3.

43. \$165 for 1y. and 8m. at 6 per ct.?
Ans. P. W. \$150. D. \$15.

44. \$121.80 for 90 days, at 6 per ct.?
Ans. P. W. \$120. D. \$1.80.

45. \$100 for 1 year at 6 per ct.?
Ans. P. W. \$$94\frac{18}{53}$. D. \$$5\frac{35}{53}$.

46. \$100 for one year at 10 per ct.?
Ans. P. W. \$$90\frac{10}{11}$. D. $9\frac{1}{11}$.

47. A note of \$500 was given, January 1, 1857, and indorsed July 4, 1857, by \$$115.33\frac{1}{3}$; another indorsement made December 25, 1857, of \$$211.33\frac{1}{3}$, what was due March 1, 1858—and if interest was paid on it then, what is due to-day, interest at 6 per ct.?
Due March 1, '58, \$$202.16\frac{2}{3}$.
Due to-day, \$———

NOTE.—Bank Discount is simple interest calculated upon amount, with "three days of grace" added to the time: this taken from the amount will leave the principal.